THE EULERIAN MULTIPHASE MODEL OF CENTRIFUGAL CASTING PROCESS OF PARTICLE REINFORCED AI MATRIX COMPOSITES

This article discusses a new model of the centrifugal casting process which uses the full Eulerian approach to modelling composite suspension dynamics. The proposed model, in contrast to models based on the Lagrangian approach, allows one to consider such important phenomena as the presence of maximum packing of reinforcing particles, or the change in viscosity suspension caused by changes in the reinforcement volume fraction. These phenomena are essential for the final result of the composite casting process. The paper presents the results of numerical experiments using the developed model. Simulation of the casting process of the AlSi7MgSr (AK7) aluminium alloy with silicon carbide and graphite was performed. The results showed that the developed model of the centrifugal casting process enables one to recognize the phenomena that are impossible to capture by measurement techniques available today.

Keywords: particle reinforced metal matrix composites (PMMCs), centrifugal casting, modelling

INTRODUCTION

Various methods both in situ [1, 2] and ex situ [3, 4] are used in order to obtain composite materials with a non-uniform, gradient or layered distribution of reinforcement in the cast. In addition to complex techniques based on infiltration processes [5, 6], the methods of powder metallurgy [7, 8] and surface modification [9], casting methods such as gravity casting [10], centrifugal [11-13] or electromagnetic casting were developed [14, 15]. The centrifugal casting of particle reinforced composites is one of the most popular methods of controlling reinforcement distribution in a cast. The method uses the density difference between the reinforcement material and the liquid matrix. Two opposing forces act on the particles: centrifugal force and buoyancy force caused by the centrifugal force acting on the matrix alloy and the pressure gradient generated by it. Depending on the difference between the density of the matrix and the reinforcement material, particle segregation may occur in the direction to the axis of rotation (higher matrix density) or in the opposite direction (higher reinforcement density). There are two basic variants of this process differing in orientation of the mould with respect to the axis of rotation: radial centrifugal casting [16] and longitudinal centrifugal casting [17]. For the first variant, radial segregation of the reinforcement was achieved. In the second variant, longitudinal casting with a suitably long arm joining the mould to the axis of rotation, approximately linear segregation of the reinforcement was obtained.

In the literature various models of the particle segregation process during the centrifugal casting of composite suspension can be found. In some of them, be-
sides, the mentioned centrifugal forces, the effect of Coriolis force was also taken into consideration [18]. The Coriolis force is particularly important in the longitudinal centrifugal casting process. Despite some differences, most of the known models are based on modeling the reinforcement dynamics by the Lagrange approach. In this approach, the movement of each particle is calculated separately, which makes it possible to analyze the trajectories of individual particles during filling of the mold. However, the modeling of independent particle motion does not allow analysis of the many important phenomena related to the interaction of reinforcement particles, such as the impact of local reinforcement concentration on the viscosity of the composite suspension, the resistance of relative motion of the particle reinforcement with their increased volume fraction, and the existence of maximum particle packing. The Eulerian model in which the dispersed phase is treated as a continuous phase does not have such limitations. For particle-reinforced composite casting models, the literature provides only two models in this approach: for gravity casting [19] and casting in an electromagnetic field [20]. This article describes the Eulerian model developed for centrifugal casting. On the basis of the proposed model, numerical simulations of casting for two composites on AlSi7MgSr matrix alloys containing different particle densities of silicon carbide and graphite particles were performed.

MODELS OF SEPARATION PROCESS OF REINFORCEMENT PARTICLES IN AI ALLOY MATRIX

Eulerian-Lagrangian model

In the Eulerian-Lagrangian approach, the matrix as a continuous phase is described by a set of two Navier-Stokes equations: the continuity equation (1) and the momentum conservation equation (2):

\[ \nabla \cdot \mathbf{v}_s = 0 \]  
\[ \frac{\partial}{\partial t} (\rho_s \mathbf{v}_s) + \nabla (\rho_s \mathbf{v}_s \cdot \mathbf{v}_s) = -\nabla P + \eta_{eff} \nabla^2 \mathbf{v}_s + \rho_s \mathbf{g} \]  

where: \( \mathbf{v}_s \) - suspension velocity, \( \rho_s \) - suspension density, \( P \) - pressure, \( \eta_{eff} \) - effective viscosity of suspension, \( \mathbf{g} \) - gravitational acceleration.

In the Lagrange approach the reinforcement phase is modeled as a discrete phase. Each particle, or group of particles having the same characteristics and trajectory are described by a set of acting forces, such as: the centrifugal force (3), the centrifugal buoyancy force (4), the gravity force (5), the buoyancy gravity force (6) and Stokes force (7) countering the movement of the particle.

\[ \mathbf{F}_c = \frac{1}{6} \pi d_p^3 \rho_p \omega^2 \mathbf{r} \]  
\[ \mathbf{F}_c = \frac{1}{6} \pi d_p^3 \rho_p \mathbf{g} \]  
\[ \mathbf{F}_{GB} = -\frac{1}{6} \pi d_p^3 \rho_m \mathbf{g} \]  
\[ \mathbf{F}_D = 3 \pi \eta_m d_p^1 (\mathbf{v}_m - \mathbf{v}_p) \]  

where: \( \rho_p \) - particle density, \( \rho_m \) - matrix density, \( d_p \) - particle diameter, \( \omega \) - angular component of velocity, \( r \) - radius vector, \( \mathbf{g} \) - gravitational acceleration, \( \eta_m \) - matrix dynamic viscosity, \( d_p \) - particle diameter, \( \mathbf{v}_p \) - particle velocity, \( \mathbf{v}_m \) - matrix velocity.

The particle motion trajectories can be determined by integrating the equation of motion over the time, taking into account the resultant of all the specified forces:

\[ \frac{d \mathbf{v}_p}{dt} = \left( \mathbf{F}_c + \mathbf{F}_{GB} + \mathbf{F}_G + \mathbf{F}_{GB} + \mathbf{F}_D \right) \frac{6}{\rho_p \pi d_p^3} \]  

Analysis of the above equations, commonly used in Lagrangian models, indicates that it is impossible to take into account the influence of the particles situated in the immediate vicinity on the movement dynamics of a specific particle. This makes it difficult or even impossible to analyze such significant phenomena for the course of the process as the change in composite suspension viscosity caused by the migration of reinforcement or the existence of a limitative share of the reinforcing phase volume fraction. It is generally accepted that the scope of the Lagrange approach is limited by the volume fraction of the dispersed phase, and does not exceed 12%. In the centrifugal separation process the volume fraction of particles is variable in the volume of the liquid matrix. In spite of the low initial volume fraction of the reinforcement phases in the composite suspension, the dynamics of centrifugal separation lead to the formation of areas in which the fraction threshold is exceeded.

Full Eulerian model

In the full Eulerian model, the reinforcement phase is not modeled as a set of separate particles, but as a continuous phase. Therefore, in this model which tracks in parallel the behaviour of two continuous phases, it is necessary to expand the system of equations describing the process in relation to the previous model. The model of phase mixture uses two separate continuity equations, both for the matrix (9) and reinforcement (10):

\[ \frac{\partial}{\partial t} (\rho_m \mathbf{v}_m) + \nabla \cdot (\rho_m \mathbf{v}_m \mathbf{v}_m) = 0 \]  

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\[
\frac{\partial}{\partial t}(\alpha_p \rho_p) + \nabla \cdot (\alpha_p \rho_p \mathbf{v}_p) = 0
\]  \tag{10}

where: \(\alpha_m\), \(\alpha_p\) - volume fraction of the matrix and reinforcement, respectively; \(\rho_m\), \(\rho_p\) - density of the matrix and reinforcement, respectively; \(\mathbf{v}_m\), \(\mathbf{v}_p\) - matrix velocity and particle velocity, respectively.

The conservation of momentum equation for an incompressible matrix takes the form:

\[
\frac{\partial}{\partial t}(\alpha_m \rho_m \mathbf{v}_m) + \nabla \cdot (\alpha_m \rho_m \mathbf{v}_m \mathbf{v}_m) = -\alpha_m \nabla p + \alpha_m \eta_m \nabla^2 \mathbf{v}_m + K_{pm}(\mathbf{v}_p - \mathbf{v}_m) + \alpha_m \rho_m \mathbf{g} + \nabla \mathbf{T}
\]  \tag{11}

where: \(p\) - common pressure to both phases, \(\eta_m\) - matrix effective viscosity, \(K_{pm}\) - momentum exchange coefficient between the phases in liquid and dispersed phases, \(\mathbf{g}\) - gravitational acceleration.

The momentum equation for the disperse reinforcement phase is described as follows:

\[
\frac{\partial}{\partial t}(\alpha_p \rho_p \mathbf{v}_p) + \nabla \cdot (\alpha_p \rho_p \mathbf{v}_p \mathbf{v}_p) = -\alpha_p \nabla p + K_{pm}(\mathbf{v}_m - \mathbf{v}_p) + \alpha_p \rho_p \mathbf{g} + \nabla \mathbf{T}
\]  \tag{12}

Equation (12) corresponds to the equation of motion of a single particle (8). The first equation term after the equal sign expresses the effect of the matrix pressure gradient on the reinforcement and is equivalent to the gravitational buoyant force (6) and the centrifugal force (4).

Another element of the equation describes the exchange of momentum between the reinforcement particles and the matrix. In contrast to the Stokes equation (7), in the Eulerian approach, we have the possibility of defining the momentum exchange coefficient \(K_{pm}\), which takes into account the influence of the local volume of reinforcement on the resistance to the motion of particles. In the developed centrifugal casting model the exchange coefficient model described by Gidaspow was used [21]. It takes into account the large volume fraction of the dispersed phase that is present in the process.

In the case of a low dispersion phase \((\alpha_p \leq 0.2)\), the exchange coefficient is described by equation (13):

\[
K_{pm} = \frac{3}{4} C_D \frac{\alpha_p \alpha_m \rho_m |\mathbf{v}_p - \mathbf{v}_m|}{\alpha_m d_p^{2.65}} \tag{13}
\]

where drag coefficient, \(C_D\) is:

\[
C_D = \frac{24}{\alpha_m Re} \left[ 1 + 0.15(\alpha_m Re)^{0.67} \right], \quad Re = \frac{\rho_m d_p |\mathbf{v}_p - \mathbf{v}_m|}{\eta_m} \tag{14}
\]

In the case of a large volume fraction of the dispersed phase \((\alpha_p > 0.2)\), the dependence describing the momentum exchange coefficient between the phases in this model is different, and it can be defined as follows:

\[
K_{pm} = 150 \frac{\alpha_p}{\alpha_m d_p^2} \frac{(1 - \alpha_m)}{\eta_m} + 1.75 \rho_m \alpha_p \left| \frac{\mathbf{v}_p - \mathbf{v}_m}{d_p} \right| \tag{15}
\]

Due to the fact that the particle volume fraction is variable during the process, the reinforcement phase can be treated as a quasi-compressive phase. For this reason, the viscous stress tensor of the dispersed phases included in the last component of the equation expresses the resistance to motion resulting from the interaction of neighbouring particles, both during shear and compression of the dispersed phase:

\[
\mathbf{T} = \alpha_p \eta_p \left[ (\nabla \mathbf{v}_p + \nabla \mathbf{v}_p^T) - \frac{2}{3} \nabla \cdot \mathbf{v}_p \mathbf{I} \right], \tag{16}
\]

where \(\mathbf{I}\) - unit tensor.

The viscosity of the dispersed phase was derived from the Dougherty-Krieg formula:

\[
\eta_p = \eta_m \left[ \frac{1 - \alpha_p}{\alpha_{max}} \right]^{-\eta_p \alpha_{max}} - 1 \tag{17}
\]

where: \(\eta_m\) - matrix effective viscosity, \(\eta_p\) - intrinsic viscosity, \(\alpha_p\) - particle volume fraction, \(\alpha_{max}\) - maximum (critical) volume fraction of particles.

The use of the viscosity equation, which takes into account the critical volume fraction, allows us to consider the maximum packing of particles phenomena in the Eulerian model. In this case the suspension viscosity approaches infinity, and blocks the possibility of exceeding the critical packing of particles during simulation.

**NUMERICAL SIMULATIONS**

The developed Eulerian model was used to simulate the casting process of an aluminium composite sleeve (AlSi7MgSr) reinforced with silicon carbide particles and graphite particles (50 µm). The initial volume fraction of the reinforcement was 10% in both cases. The cast composite sleeve had a height of 130 mm and an outer radius of 45 mm. The composite suspension was poured into a rotating mould at 3000 rpm. The pouring time of the 277 cm³ composite suspension volume was 3.5 sec.

Due to the large number of equations included in the model simulations, the numerical calculation results were very time-consuming (1 second of the process simulation took about 24 hours on a typical workstation). In order to limit the time of numerical calculation to an acceptable level, the simulations were limited to two-dimensional axisymmetric geometry.

**Simulation results for Al alloy reinforced by silicon carbide particles (SiC_p)**

The images in Figure 1 show the process of casting the AlSi7MgSr/SiC_p composite suspension into a mould.
When the liquid metal comes into contact with the bottom of the rotating mould, it is ejected onto a lateral wall surface of the mould. In the initial stage of the pouring process, there is no radial reinforcement separation. This is due to a certain delayed transfer of the rotational speed of the mould to the liquid suspension (by the action of viscosity forces) and additionally a delay of transfer of the matrix momentum to the reinforcement (Stokes resistance forces). Therefore, the centrifugal force does not act on the reinforcement yet. Since the reinforcement has a higher density than the matrix, it obtains greater momentum during the fall to the bottom of the mould. The momentum difference between both phases results in slight separation of the reinforcement along the axis of the mould during the time of 2 sec.

As a result of gathering speed by the matrix and indirectly by the reinforcement, radial separation of the reinforcement begins in 3 seconds. At the same time, the reinforcement in the lower corner of the mould begins to reach the maximum degree of packing, which results in its accumulation in this area, and consequently non-uniform, final distribution of the reinforcement after the end of the casting process.

![Fig. 1. Process of casting AlSi7MgSr/SiC₆ composite suspension into mould (black areas - matrix volume fraction, grey areas indicate presence of reinforcement)](image_url)

Rys. 1. Proces odlewania kompozytowej zawiesiny AlSi7MgSr/SiC₆ do formy (czarne obszary - udział objętościowy osnowy, szare obszary wskazują na obecność zbrojenia)
Simulation results for Al alloy reinforced by graphite particles (GRₚ)

At the time of filling the mould by the Al composite suspension with graphite particles, the process of radial reinforcement separation occurs much faster (Fig. 2). In this case, the buoyancy force is a driving mechanism and is a direct effect of matrix centrifugal motion and creating the pressure gradient. That is why the stage of momentum transfer from the matrix to the reinforcement can be ignored. This stage would be necessary if the driving force of the separation was the centrifugal force acting directly on the reinforcement. As the local reinforcement achieves the maximum degree of packing at the inner (vertical) surface of the liquid metal, the local concentration of particles is formed. This phenomenon makes it difficult to obtain a uniform reinforcement layer at the inner surface of the cast composite sleeve.

SUMMARY

The developed Eulerian model allows the phenomena present at the critical volume fraction of reinforcement during the composite casting process to be captured. This is its significant advantage in comparison to the Lagrangian models commonly used for simulation of the process.

Fig. 2. Process of casting AlSi7MgSr/GRₚ composite suspension into mould (black areas - matrix volume fraction, grey areas indicate presence of reinforcement)

Rys. 2. Proces odlewania kompozytowej zawiesiny AlSi7MgSr/GRₚ do formy (czarne obszary - udział objętościowy osnowy, szare obszary wskazują na obecność zbrojenia)
In the article, the model was used for the centrifugal casting simulation of particle reinforced metal matrix composites. However, it could be used for any other particle-reinforced composites, e.g. for polymer matrix composites or casting slip. The described model allows one to obtain knowledge of the rapidly changing processes present in the centrifugal casting process that are not possible to register with any of the currently available measurement techniques. Unfortunately, the large number of components in this model causes errors in addition resulting from some simplifications. As a result, this model cannot provide accurate quantitative data that would completely overlap the experimental data. However, it allows one to obtain knowledge of the character of the occurring phenomena. A possible way to improve the accuracy of the model seems to be to use methods for solving inverse problems in order to improve the accuracy of the model. However, it allows one to obtain knowledge of the rapidly changing processes present in the centrifugal casting process that are not possible to register with any of the currently available measurement techniques. Unfortunately, the large number of components in this model causes errors in addition resulting from some simplifications. As a result, this model cannot provide accurate quantitative data that would completely overlap the experimental data. However, it allows one to obtain knowledge of the character of the occurring phenomena. A possible way to improve the accuracy of the model seems to be to use methods for solving inverse problems in order to improve the accuracy of the model.

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