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COMPOSITES THEORY AND PRACTICE

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DESIGN OF RECTANGULAR COMPOSITE PLATES WITH CIRCULAR HOLES

The presented work is devoted to the problem of the optimal design of a multilayered composite structure. A square composite plate of geometrical dimensions 250x250x3 mm with a circular hole of diameter d = 100 mm is investigated. The structure is made of composite material, which consists of N = 12, 16 or 20 layers. Each layer is made of the same material, namely carbon fibers with epoxy resin (CFRP, fibers T300, matrix N5208). It is also assumed that the feasible fiber orientation angles are 0º, ±45º, 90º. The layer stacking sequence is symmetric with respect to the middle surface of the studied plate. The structure is subjected to uniform tension in the horizontal and vertical directions. Calculations are performed for the following load ratios, namely: p_h/p_v = 0, 0.5, 0.75, 1.0, where p_h and p_v denote the load in the horizontal and vertical directions, respectively. The optimization problem is stated as follows: we look for the stacking sequence (the number of layers with fiber orientation angle equal to 0º, ±45º, 90º, respectively), which ensures the maximal value of the load multiplier. In order to find the solution to the optimization problem, the advanced concept of the discrete design variable is introduced. The use of these variables significantly simplifies the optimization process. In consequence, a very simple optimization procedure can be utilized. All the necessary computations are carried out with the use of the commercial finite element package ANSYS 12.1. The analyzed plate is modeled as a shell structure. The optimal solution mainly depends on p_h/p_v. The total number of layers also has a slight influence on the obtained solution. The results are presented in graphs and collected in tables.

Keywords: plate with hole, optimization, stacking sequence, discrete design variables, Finite Element Method

PROJEKTOWNIE PROSTOKĄTNYCH PŁYT KOMPOZYTOWYCH Z KOŁOWYM OTWOREM

Praca poświęcona jest zagadnieniu optymalnego projektowania wielowarstwowych struktur kompozytowych. Analizie poddano kwadratową płytę o wymiarach 250x250x3 mm. Rozważana konstrukcja posiada w geometrycznym środku kołowy otwór o średnicy 100 mm. Zastosowany materiał kompozytowy składa się z N = 12, 16 lub 20 warstw. Każda warstwa wykonana jest z włókna węglowego oraz żywicy epoksydowej (CFRP, włókna T300, żywica N5208). Przyjęto także, że dopuszczalne kąty orientacji włókien w warstwach to 0º, ±45º, 90º. Rozważano jedynie konfiguracje symetryczne względem powierzchni środkowej. Analizowana płyta poddana jest równomiernemu, proporcjonalnemu rozciąganiu, zarówno w kierunku poziomym, jak i pionowym. Jako kryterium optymalizacji przyjęto maksymalną wartość mnożnika obciążenia. Optymalnej konfiguracji laminatu poszukiwano dla następujących proporcji obciążenia: p_h/p_v = 0, 0.5, 0.75, 1.0, gdzie p_h oznacza obciążenie działające w kierunku poziomym, zaś p_v w kierunku pionowym. W celu wyznaczenia rozwiązania zastosowano dyskretne zmienne decyzyjne. Podejście takie umożliwia znaczne uproszczenie procedury optymalizacji. Wszystkie niezbędne obliczenia wykonano przy wykorzystaniu komercyjnego systemu opartego na metodzie elementów skończonych - ANSYS 12.1. W wyniku przeprowadzonej analizy uzyskano optymalne konfiguracje laminatów. Rozwiązania te wyraźnie zależą od proporcji obciążenia. Niewielki wpływ na rozwiązanie ma również przyjęta liczba warstw. Uzyskane wyniki przedstawiono na wykresach oraz zebrano w odpowiednich tabelach.

Słowa kluczowe: płyta z otworem, optymalizacja, konfiguracja laminatu, dyskretne zmienne decyzyjne, metoda elementów skończonych

INTRODUCTION

Various parts of support structures contain different types of cutouts. The main disadvantage connected with the presence of holes is the stress concentrations in the vicinity of the edges of the holes. This phenomenon causes a reduction in the static strength of the structure. It is particularly important in the case of fatigue failure. In the case of cyclic load, when the applied loads are much lower than the static strength of the structure, the stress concentration leads to a rapid accumulation of damage in the composite material and results in premature failure of the whole structure. Reduction of the magnitude of stress concentration can be achieved by changing the shape of the cutout [1, 2], or by applying additional reinforcing elements [3, 4] or by the optimal selection of the fiber orientation angles in the composite material. It seems that the last proposal is the most attractive from the technological point of view. However, in the general case finding a global solution to the optimization problem is very difficult, mainly due to the large number of design variables.

These variables describe the fiber orientation angles in the particular layers. If a single design variable describes a ply angle, thus the total number of design variables will be equal to the total number of layers in the investigated composite material. This number of design variables can be significantly reduced by introducing the new idea of so-called, discrete variables. Their discrete character results due to the fact that the total number of layers is an integer. In the present work, the definition of the new design variables are introduced according the book by Muc [6].

COMPOSITE PLATE WITH CIRCULAR HOLE

The investigated square composite plate with a circular hole is shown in Figure 1. The total thickness of the plate is constant and equal to $t_c = 3$ mm. The structure is made of a composite material which consists of $N = 12$, 16 or 20 layers. The thickness of each layer is identical and equal to $t_k = t/N$. Moreover, each layer is made of the same material, namely carbon fibers with epoxy resin (CFRP, fibers T300, matrix N5208) [5]. The mechanical properties of the lamina are as follows [6]: $E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = 7.17$ GPa and $\nu_{12} = 0.28$. The admissible stresses are: $X_t = 1500$ MPa, $X_c = 1500$ MPa, $Y_t = 40$ MPa, $Y_c = 246$ MPa and $S =$ = 68 MPa, where subscript 't' and 'c' denote tension and compression, respectively and 'S' is the admissible shear stress. The minimal thickness of a lamina is equal to $t_{min} = 0.125$ mm. It is assumed that the feasible fiber orientation angles are $\theta = 0°, \pm 45°, 90°$, where angle θ is measured with respect to the X axis of the global Cartesian coordinate system (Fig. 1).

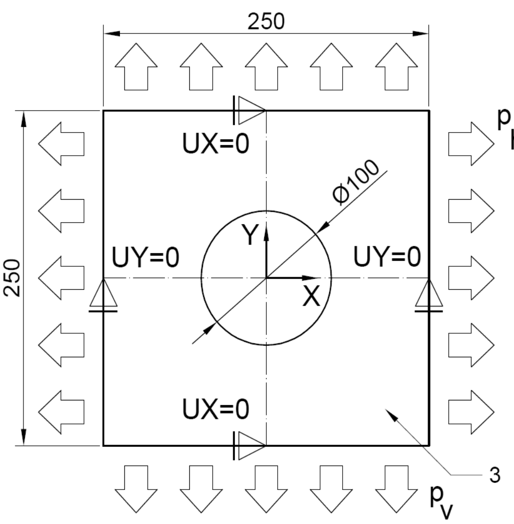

Fig. 1. Investigated composite square plate with circular hole

Rys. 1. Analizowana kompozytowa płyta z kołowym otworem

Moreover, it is also assumed that the stacking sequence is symmetric with respect to the middle surface of the plate. The exemplary feasible stacking sequence is [0°, 0°, ±45°, 90°, 90°, 90°, 90°, ±45°, 0° ,0°], or in the shorter form $[0°_2, \pm 45°, 90°_2]_S$. The investigated plate is uniformly stretched in the horizontal and vertical direction. The value of the horizontal pressure is equal to $p_h = c \cdot p_v$ (proportional system of the load, measured in [N/mm²]), where c is the proportional coefficient. In the present work $c = 0, 0.5, 0.75, 1$. If coefficient c is equal to zero, it means that the plate is stretched only in the vertical direction. The studied structure is supported in four points. The applied support allows free deformation in the vertical and horizontal directions under the abovedefined uniform load without rigid motion of the plate.

FORMULATION OF OPTIMIZATION PROBLEM

Using the classical method of design variables coding, numbers 1, 2, 3 represent the layer with the fibre orientation angle $0°_2$, ±45°, 90°, respectively. Such a representation is not very convenient for optimisation problems since there are a lot of design variables (increasing with the total number of plies N and the number of possible ply orientations). In the general case, when bending effects are also taken into consideration, various stacking sequences are described by the identical values of components of the [A], [B], [D] matrices. To simplify the analysis, the laminate stacking sequence with the following feasible fibre orientation angles: 0°, ±45°, 90°, can be represented in a specific manner proposed by Muc et al. [6, 7] (the symmetry of the stacking sequence with respect to the middle surface is also taken into account now):

$$x_r^A = \sum_{k=1}^{N/4} \cos 2\theta \, \Xi(\alpha_r), \qquad \Xi(\alpha_r) = \begin{cases} 1, & \alpha_r = \theta \\ 0, & \alpha_r \neq \theta \end{cases}, \quad \alpha_r = 90° \cdot \frac{r-1}{2}, \ r = 1, 2, 3 \tag{1}$$

Generally, x_1^A denotes the number of plies with the fiber orientation angle 0° and x_3^A denotes the number of plies with a fiber orientation angle equal to 90°. Additionally, due to the fact that the plies are in pairs, the total number of layers x_1^A, x_3^A has to be multiplied by 4. It is worth stressing here that for the fibre orientation angle $\theta = \pm 45°$, $x_2^A \equiv 0$. Thus the number of design variables is reduced and now is equal to 2. Having the values of x_1^A and x_3^A, the number of layers where the fibre orientation angle is equal to $\theta = \pm 45°$ can be determined as:

$$N_{\theta=45°} = N - 4\left(x_1^A + x_3^A\right) \tag{2}$$

Next, the components of the matrix [A] can be expressed with the use of discrete variables x_1^A and x_3^A in the following way:

$$A_{11} = t(U_1 - U_3) + \frac{4t}{N} U_2 (x_1^A - x_3^A) + \frac{8t}{N} U_3 (x_1^A + x_3^A),$$
$$A_{12} = t(U_4 + U_3) - \frac{8t}{N} U_3 (x_1^A + x_3^A),$$
$$A_{22} = t(U_1 - U_3) - \frac{4t}{N} U_2 (x_1^A - x_3^A) + \frac{8t}{N} U_3 (x_1^A + x_3^A),$$
$$A_{66} = t(U_5 + U_3) - \frac{8t}{N} U_3 (x_1^A + x_3^A), \quad (3)$$

where:

$$U_1 = \tfrac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}),$$
$$U_2 = \tfrac{1}{2}(Q_{11} - Q_{22}), \quad (4)$$
$$U_3 = \tfrac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}),$$
$$U_4 = \tfrac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66},$$
$$U_5 = \tfrac{1}{2}(U_1 - U_4),$$

and Q_{ij} are the components of the layer stiffness matrix. Here it should be stressed that in the present work, [B] ≡ 0 due to the symmetry of the stacking sequence with respect to the middle surface, and [D] ≡ 0 as only the mid-plane forces are taken into consideration.

Now, the optimization problem can be formulated as follows: we look for the stacking sequence determined by design variables x_1^A and x_3^A, which ensures the maximal value of load p_v.

$$p_v \to \max. \quad (5)$$

In order to determine the static strength of the studied plate, the criterion of admissible strain is applied. Therefore, optimization formulation (5) is equivalent to the following one in each point in the studied plate:

$$\min_{x_1^A, x_3^A} \left[\max \left(\frac{\varepsilon_1(x,y)}{\varepsilon_1^t}, \frac{|\varepsilon_1(x,y)|}{\varepsilon_1^c}, \frac{\varepsilon_2(x,y)}{\varepsilon_2^t}, \frac{|\varepsilon_2(x,y)|}{\varepsilon_2^c}, \frac{|\varepsilon_{12}(x,y)|}{\varepsilon_{12}^s} \right) \right], \quad (6)$$

where the values of the admissible strains are calculated as:

$$\varepsilon_1^t = \frac{X_t}{E_1}, \varepsilon_1^c = \frac{X_c}{E_1}, \varepsilon_2^t = \frac{Y_t}{E_2}, \varepsilon_2^c = \frac{Y_c}{E_2}, \varepsilon_{12}^S = \frac{S}{G_{12}}, \quad (7)$$

and the admissible values are $\varepsilon_1^t = 0.00829$, $\varepsilon_1^c = 0.00829$, $\varepsilon_2^t = 0.00388$, $\varepsilon_2^c = 0.02388$ and $\varepsilon_{12}^S = |0.00948|$, where subscripts '1' and '2' denote the components in the layer (local) coordinate system. It is worth noting that when the maximal value of the ratios in expression (6) is greater than unity, it means failure of the structure. Hence, taking under consideration the linear range of the performed analysis, the value of load multiplier λ_{FPF} (strength of the structure) can be estimated according to the following relationship:

$$\lambda_{FPF} = \frac{1}{\max \left(\frac{\varepsilon_1(x,y)}{\varepsilon_1^t}, \frac{|\varepsilon_1(x,y)|}{\varepsilon_1^c}, \frac{\varepsilon_2(x,y)}{\varepsilon_2^t}, \frac{|\varepsilon_2(x,y)|}{\varepsilon_2^c}, \frac{|\varepsilon_{12}(x,y)|}{\varepsilon_{12}^s} \right)} \quad (8)$$

Now, the solution to the problem can be found by testing the value of (8) in discrete points (x_1^A, x_3^A), which are related to the configuration of the composite. It is worth noting here that the layer sequence does not matter.

FINITE ELEMENT MODEL OF INVESTIGATED STRUCTURE

All the necessary computations are carried out with the use of the commercial finite element package ANSYS 12.1 Classic. The Classic module contains several types of special shell or solid elements which are particularly dedicated to the modeling of multi-layered composite structures [8, 9]. The appropriate optimization procedure was developed in the APDL script. The studied composite plate is modeled with the use of the triangular shell elements SHELL281 [9, 10]. These elements have quadratic shape functions and six degrees of freedom in each node, namely: three transitional and three rotational. They are formulated according to first order shear deformation theory. The fiber orientation angle θ is measured with respect to the direction of the axis X of the local coordinate system connected with each finite element. These coordinate systems have to be appropriately defined just before generating the finite element mesh. It is assumed that the approximate size of the finite element is equal to $l_e \approx 8$ mm. However, in the vicinity of the hole, the elements are smaller and their size is equal to $l_e \approx$ ≈ 4 mm. The FE model consists of 2446 triangular shell elements. It is worth stressing here that the assumed size of the finite elements ensures sufficient accuracy of the strain estimation in the proximity of the hole. Although the composite plate has two axes of symmetry, the whole structure is modeled. The analyzed structure is supported in the four points, as shown in Figure 1. In these points, displacement in the Z direction is also constrained.

RESULT OF OPTIMIZATION

The exemplary results of the applied optimization procedure are collected in Table 1. The presented results were obtained for the structure, where $N/4 = 5$, $p_h/p_v = 0$ and $p_v = 1.0$ N/mm^2. In the first two columns, all the combinations of the discrete design variables x_1^A, x_3^A in this case are shown. Next the corresponding values of the load multiplier λ_{FPF} are presented.

In the next three columns the fiber orientation angle of the weakest layer, the component of the strain tensor and the symbol of admissible strain value, which is active, are shown, respectively. In the last column the exemplary stacking sequence, which corresponds to the discrete design variables, is presented.

TABLE 1. **Results of optimization procedure (description in text)**
TABELA 1. **Wyniki procedury optymalizacji (opis w tekście)**

x_1^A	x_3^A	λ_{FPF}	Fiber orientation angle	Strain component	Criterion	Example of configuration
0	0	44.85311	±45°	12	ε_{12}^S	[±45°, ±45°, ±45°, ±45°, ±45°]s
1	0	41.50755	$0°_2$	2	ε_2^t	[$0°_2$, ±45°, ±45°, ±45°, ±45°]s
2	0	40.31608	$0°_2$	2	ε_2^t	[$0°_2$, $0°_2$, ±45°, ±45°, ±45°]s
3	0	34.04139	$0°_2$	2	ε_2^t	[$0°_2$, $0°_2$, $0°_2$, ±45°, ±45°]s
4	0	24.75186	$0°_2$	2	ε_2^t	[$0°_2$, $0°_2$, $0°_2$, $0°_2$, ±45°]s
5	0	13.47146	$0°_2$	2	ε_2^t	[$0°_2$, $0°_2$, $0°_2$, $0°_2$, $0°_2$]s
0	1	81.28760	±45°	12	ε_{12}^S	[$90°_2$, ±45°, ±45°, ±45°, ±45°]s
1	1	63.03183	$0°_2$	2	ε_2^t	[$90°_2$, $0°_2$, ±45°, ±45°, ±45°]s
2	1	59.66587	$0°_2$	2	ε_2^t	[$90°_2$, $0°_2$, $0°_2$, ±45°, ±45°]s
3	1	51.24001	$0°_2$	2	ε_2^t	[$90°_2$, $0°_2$, $0°_2$, $0°_2$, ±45°]s
4	1	37.47985	$0°_2$	2	ε_2^t	[$90°_2$, $0°_2$, $0°_2$, $0°_2$, $0°_2$]s
0	**2**	**108.99183**	**±45°**	**12**	$\boldsymbol{\varepsilon_{12}^S}$	**[$90°_2$, $90°_2$, ±45°, ±45°, ±45°]s**
1	2	76.75775	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $0°_2$, ±45°, ±45°]s
2	2	68.97503	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $0°_2$, $0°_2$, ±45°]s
3	2	52.72593	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $0°_2$, $0°_2$, $0°_2$]s
0	3	102.32273	±45°	2	ε_2^t	[$90°_2$, $90°_2$, $90°_2$, ±45°, ±45°]s
1	3	80.19889	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $90°_2$, $0°_2$, ±45°]s
2	3	64.16014	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $90°_2$, $0°_2$, $0°_2$]s
0	4	76.05141	±45°	2	ε_2^t	[$90°_2$, $90°_2$, $90°_2$, $90°_2$, ±45°]s
1	4	72.27000	$0°_2$	2	ε_2^t	[$90°_2$, $90°_2$, $90°_2$, $90°_2$, $0°_2$]s
0	5	67.41725	$90°_2$	12	ε_{12}^S	[$90°_2$, $90°_2$, $90°_2$, $90°_2$, $90°_2$]s

As can be observed for p_v = 1 N/mm², the layers with the fiber orientation angle $\theta = 0°$ are the weakest. If there is no such a layer in the composite material, failure of the layer is first observed in the case of $\theta = ±45°$. The strongest layers are those where the angle $\theta = 90°$. Failure of the layer is caused mainly by reaching admissible tension strain (ε_2^t) in the direction perpendicular to the fiber. However, in the case of several configurations, the admissible shear strain (ε_{12}^S) is also reached. The optimal structure (bold row in Table 1) is obtained for the following discrete design variable, namely: $x_1^A = 0$ and $x_3^A = 2$, which corresponds to the following exemplary stacking sequence: [$90°_2$, $90°_2$, ±45°, ±45°, ±45°]s. Similar calculations were carried out for 12 composite plates. All the studied plates have an identical total thickness (t_c = 3 mm). The obtained results of optimization are shown in Figures 2 and 3. The maximal values of load multiplier are collected in Table 2.

In the case of a load when $p_h/p_v = 0$ (uniform tension in vertical direction only), the best solution is obtained for the stacking sequence where there are no layers with the fiber orientation angle $\theta = 0°$ ($x_1^A = 0$). The optimal stacking sequence consists of 4 or 8 layers with the fiber orientation angle $\theta = 90°$ (Table 2).

The rest of the layers have $\theta = ±45°$. The maximal value of load multiplier slightly depends on the number (thickness) of layers. In can be found that the strongest structure is obtained for the total number of layers $N = 16$ ($N/4 = 4$) and then λ_{FPF} = 111.869 MPa. It is worth stressing here that there is a significant difference between the strongest and the weakest structure. The maximal value of load multiplier λ_{FPF} is over 8 times greater in comparison with the minimal one. As can be observed in Figure 2, the minimal value of load is obtained for the stacking sequence where all the layers have θ = 0°. Examples of the optimal stacking sequences are [$90°_2$, ±45°, ±45°]S, [$90°_2$, $90°_2$, ±45°, ±45°]S or [$90°_2$, $90°_2$, ±45°, ±45°, ±45°]S.

TABLE 2. **Values of maximal load λ_{FPF}**
TABELA 2. **Maksymalne wartości obciążenia λ_{FPF}.**

	Number of layers								
	$N/4 = 3$			$N/4 = 4$			$N/4 = 5$		
P_h/p_v	λ_{FPF}	x_1^A	x_3^A	λ_{FPF}	x_1^A	x_3^A	λ_{FPF}	x_1^A	x_3^A
0.00	100.412	0	1	111.869	0	2	108.992	0	2
0.500	131.366	0	1	126.225	0	1	129.475	0	2
0.75	113.210	0	1	119.931	0	1	121.543	1	2
1.00	113.150	1	1	120.013	1	1	120.430	1	1

Total thickness of composite plate t_c = 3 mm.

In the case of $p_h/p_v = 0.5$, the obtained optimal stacking sequences are similar to those which are presented above. However, the maximal values of load multiplied λ_{FPF} are significantly greater in comparison with those which are presented for $p_h/p_v = 0$. It is worth noting that the trajectories shown in Figure 2 are also slight different. It is caused by the presence of horizontal tension.

In the next case (p_h/p_v = 0.75), further influence of the increasing horizontal tension can be observed (Fig. 2). Now, in the optimal stacking sequence for the case when $N/4$ = 5, there are 4 layers with the fiber orientation angle θ = 0° and it corresponds to the change in the optimum locaation in the discrete design variables space. Moreover, the values of maximal load are much lower in comparison with the previous case.

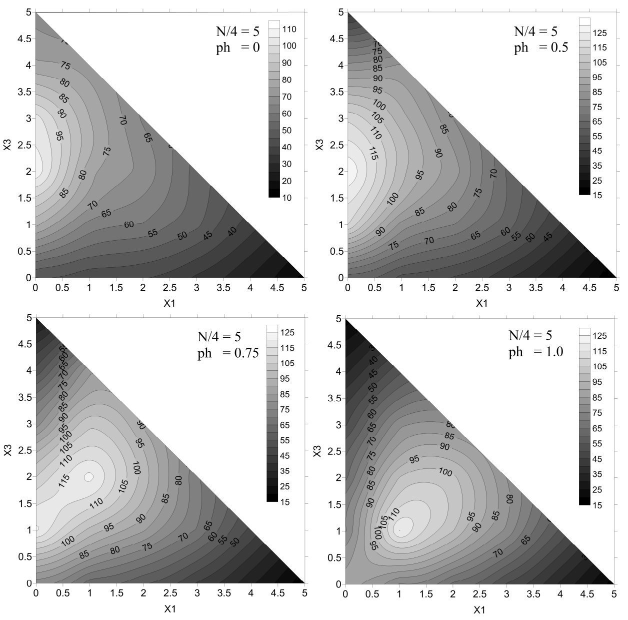

Fig. 2. Design space (x_1^A , x_3^A) with results of optimization for $N/4$ = 5

Rys. 2. Obszar zmiennych decyzyjnych (x_1^A , x_3^A) z zaznaczonymi wynikami optymalizacji dla $N/4$ = 5

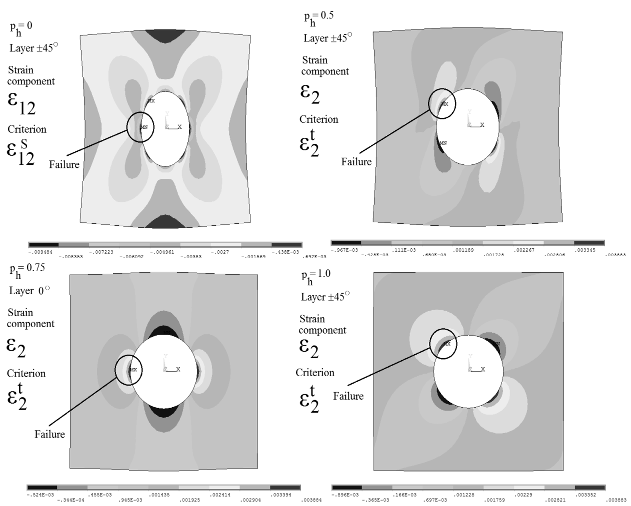

Fig. 3. Distribution of strain component for which admissible value is exceeded ($N/4$ = 5)

Rys. 3. Rozkład składowych odkształcenia dla których przekroczono dopuszczalną wartość ($N/4$ = 5)

In the last studied case, when the composite plate is uniformly stretched in both directions, the optimal stacking sequences are very similar for all the numbers of layers. Each optimal configuration consists of 4 layers with the fiber orientation angle θ = 0° and 4 with θ = 90°. The graph which corresponds to this case in Figure 2, is regular and symmetric. A single global optimum can be observed in this case. The values of the maximal load multiplier λ_{FPF} are almost identical in comparison with the previous case.

Finally, in Figure 3 the strain distribution in the weakest layers (optimal structures, $N/4$ = 5) in each studied case of load is presented. In the case of p_h = 0, the weakest layer has the fiber orientation angle θ = ±45°. The failure is caused by exceeding the admissible value of the ε_{12}^S strain component. The layers in the case of p_h = 0.5, 10 have the same fiber orientation angle. Nevertheless in these cases, failure of the structure is caused by exceeding the admissible value of the ε_2^t (tension) strain component. As may be observed, the location of the failure around the hole varies. For the isotropic plates, the failure occurs in the perpendicular direction to the loading direction (p_h = 0). For uniform loading (p_h = 1), the failure occurs uniformly around the hole. In Figure 3, the position of maximal strain is different than the isotropic materials.

CONCLUSIONS

It is worth stressing here that introducing the new concept of design variables significantly simplifies the optimization process in the case of multilayered composite materials. In order to find the solution to the problem, a simple algorithm is applied. This algorithm is based on testing the values of the load multiplier in the finite set of discrete points, namely x_1^A , x_3^A. Generally, the optimal composite material consists of layers with the fiber orientation angle θ = ±45°. If the horizontal component of load is absent, the optimal stacking sequence is complemented by layers with the fiber orientation angle θ = 90°. If the horizontal component of load is present and if its value is equal to the vertical component of load ($p_v = p_h$), the optimal stacking sequence consists of an identical number of layers with fiber orientation angles θ = 0° and θ = 90°. The total number of layers has little influence on the optimal value of the load multiplier.

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