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INFLUENCE OF SELECTED METHOD TO ESTIMATE COMPOSITE MATERIAL ELASTICITY PROPERTIES ON RESULTS OF FINITE ELEMENT ANALYSIS

To perform accurate finite element analysis of composite materials, it is a necessity to enter the correct value of the material properties during model preparation. Since composites consist of at least two components with quite different properties, it is necessary to calculate the resultant properties of the final material. There are a few methods for estimating these parameters. This paper focused on two methods referred to as the Rule of Mixture model, and the Halpin-Tsai model. In the first section, the basic elastic properties of the component materials were cited, then in the next part of paper the elastic properties of the composites were calculated using the aforementioned method. During the calculations both methods were described, along with the range of their application in reality. The parameters were calculated for several composites differing in the percentage of fibers in the composite to present changes in the coefficients on a common graph. All the moduli are collected and presented on graphs. One material was then chosen and used for further analysis. In the next step, finite element analysis (FEA) was performed to estimate the impact of the selected method for determining the material elastic properties on the results of the analysis. FEA analysis was performed using ANSYS Workbench 19.1 ACP Pre/Post software, which allows users to manually enter the properties of the studied materials. The sample was modelled as a laminate, consisting of layers - laminas arranged angularly in relation to each other, which are arranged symmetrically with respect to the median plane of the sample. Two analyses were carried out and then the differences in the final results of the strain values were presented in table form, which indicate the percentage differences between them. This allows one to specify which model of estimating the elastic properties is more precise in the studied case and when it is worth using a more precise method of estimating these values.

Keywords: material properties, composite materials, Young modulus for composites, finite element method

WPŁYW WYBRANEJ METODY OSZACOWANIA WARTOŚCI WSPÓŁCZYNNIKÓW SPRĘŻYSTOŚCI MATERIAŁU KOMPOZYTOWEGO NA WYNIKI ANALIZY METODĄ ELEMENTÓW SKOŃCZONYCH

Do przeprowadzenia dokładnej analizy z wykorzystaniem metody elementów skończonych dla materiałów kompozytowych konieczne jest wprowadzenie podczas przygotowania modelu odpowiedniej wartości dla współczynników charakteryzujących materiał. Ponieważ kompozyty składają się z minimum dwóch składników, różniących się między sobą właściwościami, konieczne jest oszacowanie parametrów wynikowych. Istnieje kilka metod pozwalających na oszacowanie tych parametrów. Artykuł skupia się na dwóch modelach określanych odpowiednio jako Rule of Mixture oraz Halpin-Tsai. Na samym początku przytoczono podstawowe właściwości sprężyste materiałów składowych, następnie wyliczono właściwości sprężyste kompozytów z wykorzystaniem wspomnianych wcześniej metod. Tymi parametrami są: moduł elastyczności wzdłużnej, moduł elastyczności poprzecznej, współczynnik Poissona, moduł ścinania w płaszczyźnie XY. Podczas obliczeń zostały opisane obie metody wraz z zakresem ich zastosowania w rzeczywistości. Parametry obliczono dla kilku kompozytów różniących się procentowym udziałem włókien w kompozycie w celu zaprezentowania zmian współczynników na wspólnym wykresie. Następnie jeden materiał wybrano do przeprowadzenia dalszej analizy. W kolejnym kroku przeprowadzono analizę z wykorzystaniem MES w celu oszacowania wielkości wpływu dobranej metody określania własności sprężystych materiału na wyniki analizy. Do przeprowadzenia analizy użyto oprogramowania ANSYS Workbench 19.1 ACP Pre/Post, które pozwala na ręczne wprowadzenie parametrów nowo utworzonego materiału kompozytowego. Próbka została zamodelowana jako laminat, składający się z warstw - lamin ułożonych kątowo w stosunku do siebie, które rozmieszczone są symetrycznie względem płaszczyzny środkowej próbki. Przeprowadzone zostały dwie analizy, które pozwoliły zaprezentować różnice w wydłużeniach próbek. To pozwala na sprecyzowanie, który model oszacowania parametrów elastycznych jest bardziej dokładny w badanym przypadku oraz kiedy warto używać bardziej precyzyjnej metody szacowania tych wartości.

Słowa kluczowe: właściwości materiałowe, materiały kompozytowe, moduł Younga dla kompozytów, metoda elementów skończonych

INTRODUCTION

Polymer composites consist of at least two independent components which differ from each other in the area of material and mechanical properties. Each composite structure contains a matrix, which connects and protects the fibers from external impacts, give proper shape and transfer loads between the fibers. The other part of the composite structure is the reinforcement. It is necessary to choose proper reinforcement during the design process, which means the correct material, shape of reinforcing elements, the volume fraction etc. The resultant properties of the newly created composite structure, which can be considered on the macro-scale, depend on the properties of each component material, the shape of the reinforcing elements and the volume fractions [1]. A composite laminate which consists of layers made as unidirectional glass fiber laminas saturated with epoxy resin is taken into consideration. The layers in this laminate are arranged at appropriate angles to each other. Micromechanics allow one to predict the behavior of a composite, based on the interaction between the components on the microscale (matrix - fibers) to define their influence on the newly created composite. All the presented methods assume that the bonds between the matrix and the fibers are perfect and there are no additional inclusions in the composite.

Component material properties

All the necessary properties of the component materials which are used in further analyses are presented in Tables 1 and 2.

TABLE 1. **Mechanical properties of epoxy resin [2]**
TABELA 1. **Właściwości mechaniczne dla żywicy epoksydowej [2]**

Density	Tensile modulus E	Poisson's ratio ν	Shear modulus G	Longitudinal tensile strength
1540 kg/m^3	3.5 GPa	0.33	1.25 GPa	60 MPa

TABLE 2. **Mechanical properties of glass fibers [2]**
TABELA 2. **Właściwości mechaniczne dla żywicy epoksydowej [2]**

Density	Longitudinal tensile todulus E_1	Transverse tensile modulus E_2	Poisson's ratio ν_{12}	Shear modulus G_{12}	Longitudinal tensile strength
2450 kg/m^3	71 GPa	71 GPa	0.22	30 GPa	3500 MPa

Different proportions of the components presented above were used to evaluate the mechanical properties of a few newly made composite structures. Each modulus was calculated for several variants of fiber content in the lamina (35÷75%) and the final results are presented in Tables 3 and 4.

CALCULATING COMPOSITE PROPERTIES

The elastic properties of laminate made from unidirectionally fiber-reinforced composite material were calculated. These properties consists of the following moduli and ratio of the created composite: E_1 - longitudinal modulus, E_2 - transverse modulus, ν_{12} - major Poisson's ratio and G_{12} - in-plane shear modulus.

During the calculations, two independent methods were used to estimate the properties of the newly created composite. These methods are the rule of mixtures (RoM) and the Halpin-Tsai model (H-T).

The rule of mixtures is one of the most basic models used to predict composite properties based on the matrix properties, fiber material and fiber volume fraction. It does not take into account the fiber shape and fiber distribution. Furthermore, a perfect bond between the fibers and the matrix is assumed. It allows one to estimate a very precise longitudinal Young modulus E_1 and Poisson's ratio, however, the predictions of the transverse modulus are underestimated and it is good practice to use another type of model to predict their correct values [3].

The Halpin-Tsai model is based on the existing scale factor - ξ, which can be used to fit the model to the experimentally determined values of the properties. This factor allows one to scale the model to any fiber arrangements, fiber shape or bond efficiency between the matrix and the fibers. By using the scale factor the model can be also stiffened or weakened transversally to the fiber directions, which makes it really universal.

Longitudinal modulus E_1

The first determined modulus refers to fiber direction of the lamina, thus in one direction according to Figure 1. There is an assumption that both the matrix and fibers present the same strains in the fiber direction of unidirectional fiber reinforced material [4].

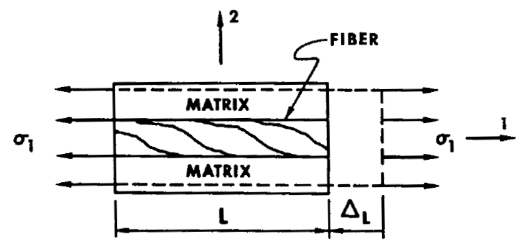

Fig. 1. Representative volume element loaded along Direction 1 [5]
Rys. 1. Reprezentatywny element objętościowy obciążony wzdłuż osi włókien - kierunek 1 [5]

To calculate this value we use Equation (1) which is common for the RoM and Halpin-Tsai method.

Longitudinal modulus E_1:

$$E_1 = E_f \cdot V_f + E_m \cdot V_m \quad (1)$$

where: E_1 - longitudinal Young modulus, E_f - fiber Young modulus, V_f - fiber volume fraction, E_m - matrix Young modulus, V_m - matrix volume in composite.

As result, we get a simple linear variation of Young modulus E_1, presented in Figure 2a. Fiber modulus E_f is around 20 times larger than E_m, thus E_1 is mostly based on the E_f value.

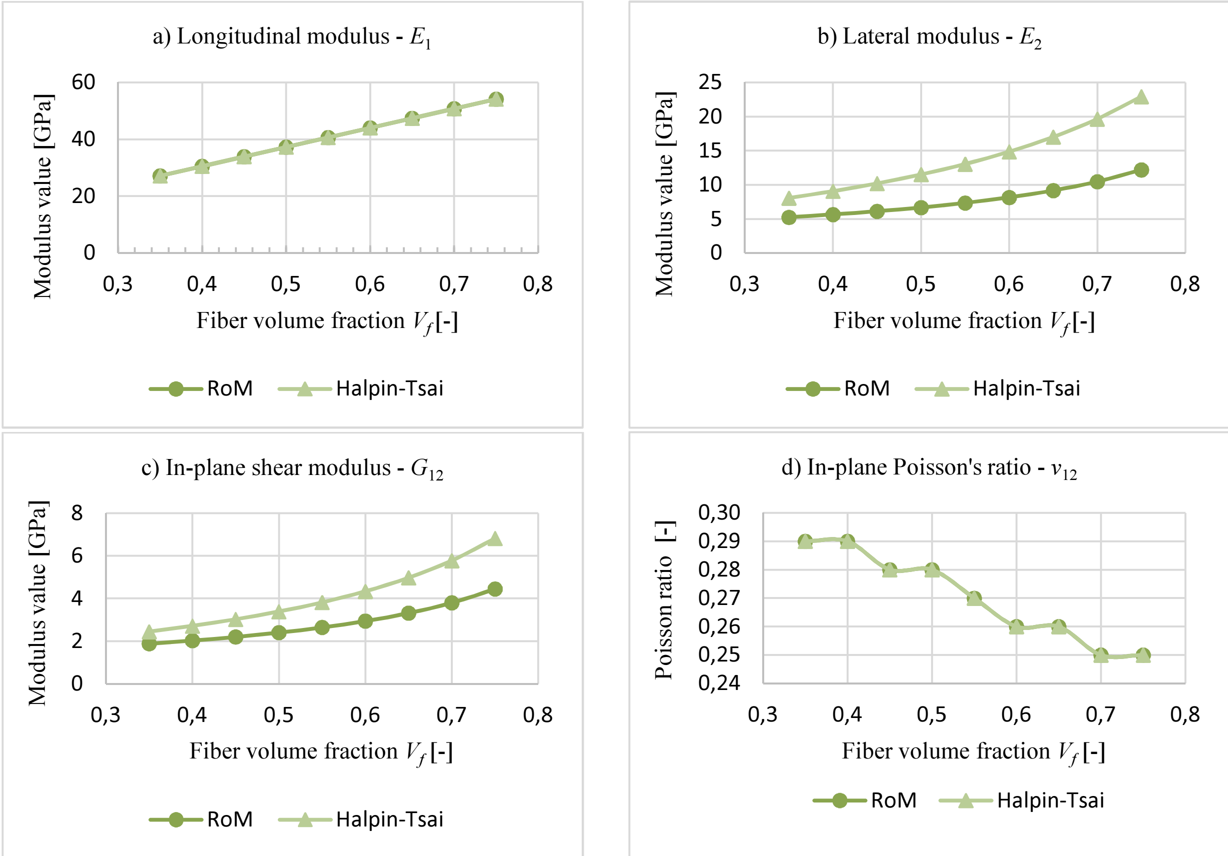

Fig. 2. Calculated moduli values

Rys. 2. Obliczone wartości modułów

Transverse modulus E_2

The second modulus is Young modulus E_2 with a direction transverse to the fibers. There is an assumption that the same stresses σ_2 are applied to the fiber and the matrix, but there is no assumption about the same strains in the fiber and matrix as in the previous modulus [6].

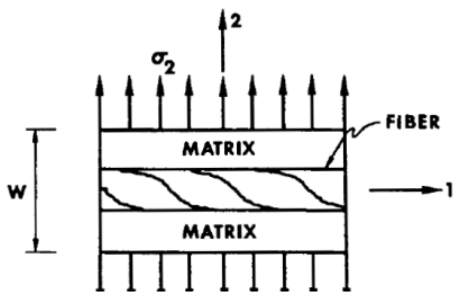

Fig. 3. Representative volume element loaded along Direction 2 [5]

Rys. 3. Reprezentatywny element objętościowy obciążony poprzecznie do osi włókien - kierunek 2 [5]

To calculate the transversal module we use Equation (2) for the RoM method and Equation (3) for the Halpin-Tsai method. Transverse modulus E_2 - rule of mixtures:

$$E_2 = \frac{E_f \cdot E_m}{E_f \cdot V_m + E_m \cdot V_f} \quad (2)$$

where: E_2 - transverse Young's modulus, E_f - fiber Young modulus, V_f - fiber volume fraction, E_m - matrix Young modulus, V_m - matrix volume in composite.

Transverse modulus E_2 - Halpin-Tsai:

$$\frac{E_2}{E_m} = \frac{1+\xi\eta V_f}{1-\eta \cdot V_f} \quad (3)$$

$$\eta = \frac{\frac{E_f}{E_m}-1}{\frac{E_f}{E_m}+\xi}$$

where: E_2 - transverse Young modulus, E_f - fiber Young modulus, V_f - fiber volume fraction, E_m - matrix Young modulus, $\xi = 2$.

To calculate the E_2 modulus, we assume that $\xi = 2$, which gives good results for circular fibres in a square array [7]. Based on Figure 2b we can see that the modulus calculated using the Halpin-Tsai method is higher than the one calculated with RoM, and with increasing the volume fraction, the difference increases as well.

In-plane shear modulus G_{12}

When calculating the G_{12} shear modulus of the lamina, the same shearing stresses for the fiber and the matrix were assumed [8]. This load is shown in Figure 4. The shear deformations are not the same because the matrix deforms more that the fibers during shear because of its lower shear modulus.

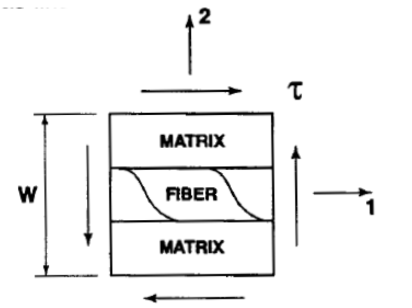

Fig. 4. Representative volume element loaded in shear [5]

Rys. 4. Reprezentatywny element objętościowy obciążony na ścinanie [5]

Calculating G_{12} according to the ROM and H-T equations looks similar to the E_2 modulus calculation.

In-plane shear modulus G_{12} - rule of mixtures:

$$G_{12} = \frac{G_f \cdot G_m}{G_f \cdot V_m + G_m \cdot V_f} \tag{4}$$

where: G_{12} - in-plane shear modulus, G_f - fiber Young modulus, V_f - fiber volume fraction, G_m - matrix Young modulus, V_m - matrix volume in composite.

In-plane shear modulus G_{12} - Halpin-Tsai:

$$\frac{G_{12}}{G_m} = \frac{1+\xi \eta V_f}{1-\eta \cdot V_f} \tag{5}$$

$$\eta = \frac{\frac{G_f}{G_m}-1}{\frac{G_f}{G_m}+\xi}$$

where: G_{12} - in-plane shear modulus, G_f - fiber Young modulus, V_f - fiber volume fraction, G_m - matrix Young modulus, $\xi = 1$.

To calculate the G_{12} modulus, we assume that $\xi = 1$, which gives the best results for the fiber volume fraction at the level of 55% [5].

Major Poisson's ratio υ_{12}

Poisson's ratio determines the effect of deformations related to the direction of normal load on deformations arising in the direction transverse to the direction of loads. It is expressed by the ratio of linear deformations and, for example, in Figure 5, it can be obtained using assumptions similar to those for determining the modulus of elasticity [4].

To calculate the in-plane Poisson ratio we use Equation (6) which is common for the RoM and Halpin-Tsai methods:

$$\upsilon_{12} = \upsilon_f \cdot V_f + \upsilon_m \cdot V_m \tag{6}$$

where: υ_{12} - longitudinal Young modulus, υ_f - fiber Young modulus, V_f - fiber volume fraction, υ_m - matrix Young modulus, V_m - matrix volume in composite.

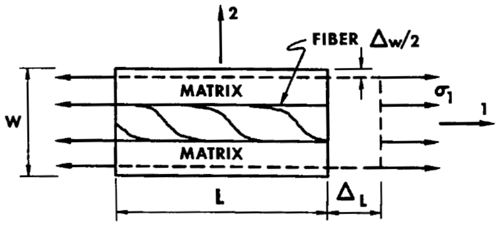

Fig. 5. Representative volume element loaded along Direction 1 [5]

Rys. 5. Reprezentatywny element objętościowy obciążony wzdłuż osi włókien - 1 [5]

A summary of the results of all the calculated ratios is presented in Tables 3 and 4. The composite material with the fiber volume equal to 55% was selected to perform detailed FEA.

TABLE 3. **Mechanical properties estimated using rule of mixtures method**

TABELA 3. **Właściwości mechaniczne oszacowane z użyciem metody rule of mixtures**

V_f [%]	35	40	45	50	55	60	65	70	75
E_1 [GPa]	27.13	30.50	33.88	37.25	40.63	44.00	47.38	50.75	54.13
E_2 [Gpa]	5.25	5.65	6.12	6.67	7.34	8.15	9.16	10.46	12.20
G_{12} [GPa]	1.88	2.03	2.20	2.40	2.64	2.94	3.31	3.80	4.44
υ_{12} [-]	0.29	0.29	0.28	0.28	0.27	0.26	0.26	0.25	0.25

TABLE 4. **Mechanical properties estimated using Halpin-Tsai equations**

TABELA 4. **Właściwości mechaniczne oszacowane z użyciem metody Halpin-Tsai**

V_f [%]	35	40	45	50	55	60	65	70	75
E_1 [GPa]	27.13	30.50	33.88	37.25	40.63	44.00	47.38	50.75	54.13
E_2 [Gpa]	8.06	9.06	10.20	11.51	13.04	14.84	17.00	19.63	22.92
G_{12} [GPa]	2.44	2.71	3.02	3.38	3.81	4.33	4.97	5.77	6.81
υ_{12} [-]	0.29	0.29	0.28	0.28	0.27	0.26	0.26	0.25	0.25

FEM CALCULATION

The calculation was performed using the ACP module in ANSYS Workbench 19.1 software. The composite model was prepared using ACP Pre, and then, after the calculations, the results for each ply were presented using ACP Post.

Sample, boundary conditions and calculation parameters

The tests were performed with a composite sample whose dimensions are 40x160x1.6 mm. The sample was made using E-glass plies which were laid in the following way: $[90 / 0_2 / 45]_s$. The exact elastic material properties were calculated with the rules of mixtures and Halpin-Tsai models. The stress and strain limits for the materials were kept the same during all the tests. The applied force is presented in Figure 6 as the displacement of the B surface whose value equals 0.8 mm in the "X" direction.

The parameters developed to prepare the correct mesh are: Type - Element size, Element Order - Quadratic, Element Size - 5 mm.

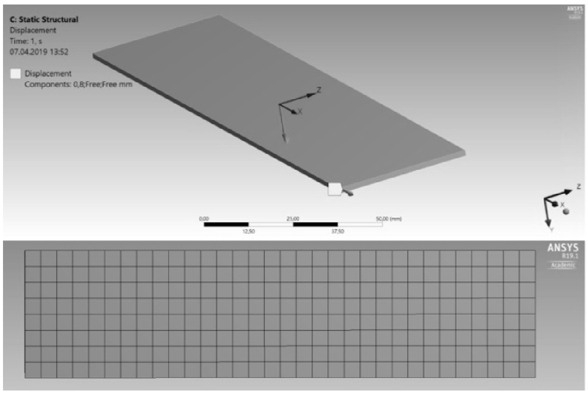

Fig. 6. Boundary conditions of test sample and visualization of sample after meshing

Rys. 6. Warunki brzegowe dla testowanej próbki oraz prezentacja modelu po nałożeniu siatki

The elastic properties of the composite materials calculated in previous sections were entered in the ANSYS software using the Engineering Data database. The materials were added to the database as a prepreg, hence they already consist of composed fibers and resin. When preparing the model in ACP Pre, it is not necessary to add an additional adhesive layer between the laminas. The failure criteria which were selected during the analyses are the "Maximum Stress" and "Maximum Strain".

In the maximum strain criterion, the ratios of current strains to the failure strains are compared in the ply principal coordinate system. This function can be described as [9]:

$$f = max\left(\left|\frac{\varepsilon_1}{X_\varepsilon}\right|, \left|\frac{\varepsilon_2}{Y_\varepsilon}\right|, \left|\frac{\varepsilon_3}{Z_\varepsilon}\right|, \left|\frac{\gamma_{12}}{S_\varepsilon}\right|, \left|\frac{\gamma_{13}}{R_\varepsilon}\right|, \left|\frac{\gamma_{23}}{Q_\varepsilon}\right|\right) \quad (7)$$

where:

$$\varepsilon_1 \geq 0 \rightarrow X_\varepsilon = X_{\varepsilon t};\ \varepsilon_1 < 0 \rightarrow X_\varepsilon = X_{\varepsilon c}$$
$$\varepsilon_2 \geq 0 \rightarrow Y_\varepsilon = Y_{\varepsilon t};\ \varepsilon_2 < 0 \rightarrow Y_\varepsilon = Y_{\varepsilon c}$$
$$\varepsilon_3 \geq 0 \rightarrow Z_\varepsilon = Z_{\varepsilon t};\ \varepsilon_3 < 0 \rightarrow Z_\varepsilon = Z_{\varepsilon c}$$

In the maximum stress criterion the ratios of current stresses to the failure strains are compared in the ply principal coordinate system. This function can be described as [9]:

$$f = max\left(\left|\frac{\sigma_1}{X}\right|, \left|\frac{\sigma_2}{Y}\right|, \left|\frac{\sigma_3}{Z}\right|, \left|\frac{\tau_{12}}{S}\right|, \left|\frac{\tau_{13}}{R}\right|, \left|\frac{\tau_{23}}{Q}\right|\right) \quad (8)$$

where:

$$\sigma_1 \geq 0 \rightarrow X = X_t;\ \sigma_1 < 0 \rightarrow X = X_c$$
$$\sigma_2 \geq 0 \rightarrow Y = Y_t;\ \sigma_2 < 0 \rightarrow Y = Y_c$$
$$\sigma_3 \geq 0 \rightarrow Z = Z_t;\ \sigma_3 < 0 \rightarrow Z = Z_c$$

Rule of mixtures variant

Maximum strains are observed in the ply which is situated 90^o in relation to the force direction and their maximum value is about 1.9298. Failure in this case may occur due to exceeding the criterion of maximum permissible strain, which is indicated as e2t on the model presenting the results of the analysis, which means e2t exceeded the strain in 2nd material direction because of tension according to the failure criteria.

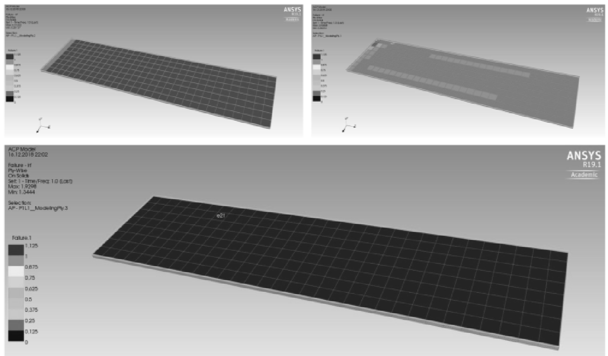

Fig. 7. Results from FEA of spample with properties calculated using rule of mixtures model. Results divided by plies (0^o, 45^o, 90^o)

Rys. 7. Graficzna prezentacja wyników analizy MES próbki dla właściwości obliczonych wg modelu rule of mixtures. Wyniki przedstawione osobno dla poszczególnych warstw (0, 45^o, 90^o)

Halpin-Tsai variant

In the analysis which was based on the Halpin-Tsai model of estimating the elastic properties, maximum strains exist in the same ply as in the previous analysis (90^o), but in this case the maximum strains are equal to 2.33. Failure may occur due to exceeding the criterion of maximum permissible stress as s2t, which means s2t exceeded the stress in the 2nd material direction because of tension according to the failure criteria.

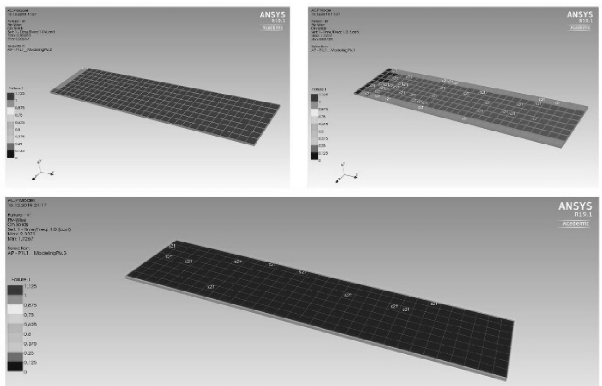

Fig. 8. Results from FEA of sample with properties calculated using Halpin-Tsai model. Results divided by plies (0^o,45^o,90^o)

Rys. 8. Graficzna prezentacja wyników analizy MES próbki dla właściwości obliczonych wg modelu Halpin-Tsai. Wyniki przedstawione osobno dla poszczególnych warstw (0, 45^o, 90^o)

Results comparison

Besides the differences in the maximum strains between the rule of mixtures and the Halpin-Tsai models in the layer angled 90^o to main force direction, there is also a difference in the 0 and 45^o plies. According to Figures 7 and 8, the maximum strain is 0.97 in RoM and 1.72 in HT. It may be considered a huge difference which may have an impact on the conclusions after the analysis.

CONCLUSIONS

Calculating the elastic properties using the rule of mixtures model is more convenient, however, it is necessary to be aware of simplifications when estimating the Young modulus value with the help of this model. Based on tests performed as experiments with a real test sample presented in publication [3], the results indicate that longitudinal models using either the RoM or HT method give very good predictions in relation to the experimental value. However, transverse to the fiber direction, RoM underestimated the properties of the transverse modulus as well as the in-plane shear modulus [3].

The Halpin-Tsai method allows one to fit the model to the experimental value using the reinforcing parameter - ξ. The most reliable way is to perform real tests and then fit the model using “ξ" to receive more accurate results. By using a reinforcing parameter value of 0, the model equals the transverse value of the RoM model, but if the reinforcing parameter is going to be infinite, then model equals the longitudinal value of the RoM. A further advantage is the possibility to adjust the ratio to the shape of the fibers or the bonding efficiency between fibers and matrix. Based on that, the properties calculated using the Halpin-Tsai method are more precise but more complicated. The values are more accurate to those received during real tests.

Comparison of the presented results from the FEA show differences in the strain levels in the respective layers of both samples. The percentage values are presented in Table 5. The strains for all the plies are larger using the Halpin-Tsai estimation than the rule of mixtures.

TABLE 5. **Percentage differences in strain in respective layers**
TABELA 5. **Procentowa różnica w wydłużeniach dla odpowiednich warstw**

[−]	Maximum strain value		
	0°	45°	90°
Rule of mixtures	0.51	0.97	1.93
Halpin-Tsai	0.90	1.72	2.33
Percentage differences	76%	56%	21%

As was indicated before, the material properties calculated using the H-T method are more accurate than those evaluated with the RoM, moreover, the maximum strain value is much larger than in the RoM method, therefore using the H-T method is safer for FEA. It is more reliable to obtain a higher value of strains in the material and then strengthen them than receive underestimated results which are not accurate enough.

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