Polish Society of Composite Materials
Polskie Towarzystwo Materiałów Kompozytowych

22: 4 (2022) 225-232

COMPOSITES THEORY AND PRACTICE
ISSN: 2084-6096
ISSN (online): 2299-128X

Abdelhak Khechai[1]*, Abdelouahab Tati[2], Mohamed-Ouejdi Belarbi[1], Fares Mohammed Laid Rekbi[3]

[1] *Laboratoire de Recherche en Génie Civil, LRGC, Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria*
[2] *Laboratoire de Génie Energétique et Matériaux, LGEM, Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria*
[3] *Research Center in Industrial Technologies CRTI, P.O. Box 64, Cheraga 16014, Algeria*
**Corresponding author. E-mail: a.khechai@univ-biskra.dz*

Received (Otrzymano) 7.09.2022

STRESS AND FAILURE ANALYSIS OF COMPOSITE PLATES WITH A CIRCULAR HOLE SUBJECTED TO SHEAR LOADING. PART 2: RESULTS AND DISCUSSIONS

This paper, the second part of two parts of a complete paper, presents the analytical and numerical results of stresses around circular cutouts in anisotropic and isotropic plates under shear loading. The main aim of this study is to understand the effect of the presence of cutouts on the stress concentration and failure mechanisms in composite laminates. The numerical investigations are performed by means of the quadrilateral finite element of four nodes with thirty-two degrees of freedom. The present finite element is a combination of two finite elements. The first one is a simple linear isoparametric membrane element and the second one is a high-precision rectangular Hermitian element. The analytical and finite element formulations were presented in the first part of the paper. Several new examples are considered to demonstrate and affirm the accuracy and the performance of the present element and to highlight the effect of some parameters on the stress distributions. The numerically obtained results are found to be in good agreement with the analytical findings. On the other hand, first ply failure (FPF) strengths in laminates with and without holes are calculated by adapting the Hashin-Rotem, Tsai-Hill, and Tsai-Wu failure theories. Finally, the numbers of the figures are obtained, using various E_1/E_2 ratio values, for the maximum positive and negative stresses values located in the vicinity of the cutout versus the angular location of points, and for various fiber orientation angles.

Keywords: **anisotropic plates, stress concentration, circular cutout, failure criterion, shear loading**

INTRODUCTION

Composite materials have widespread applications in aerospace and other industries where weight reduction and directional properties are the main criteria [1]. Circular cutouts are unavoidable in these materials to satisfy the needs of the design. Cutouts change the mechanical behavior of structures and produce a highly undesirable stress concentration around these cutouts [2-8]. The accurate design of laminated plates with notches requires the stress distribution to be determined by means of appropriate methods [9]. Stress analysis has received much attention from many researchers, who have carried out their studies using different analytical and numerical approaches [10-13]. Significant progress in analytical solutions for anisotropic structures with cutouts was achieved by Lekhnitskii [14] and Savin [15]. These methods essentially generalize the Muskhelishvili [16] method for resolving two-dimensional problems of isotropic materials with complex stress analysis. Some recent references for the contemporary application of these analytical methods can be found in [17-21].

Recently, Dharmin et al. [22] and Nagpal et al. [23] published review papers regarding recent analytical methods and numerical techniques for the stress analysis of composite laminated plates with cutouts. As a result, only few investigations on laminated plates under shear loading were found compared to numerical investigations on plates under tensile loading. Thus, most research studies have been performed employing analytical approaches based on Lekhnitskii [14] or Savin [15], whereas the only in-depth research work based on the theoretical elasticity of Green and Zerna [24] was published by Arslan [25]. His study proposed a very simple analytical solution to determine the stress distribution in single-layer plates. Arslan [25] studied the fiber orientation angle effect on the stress values located in the vicinity of circular cutouts in single-layers, and their failure loads using Tsai-Hill [26] and Hencky-von Mises failure criteria. On the other hand, to determine the stress field in composite material, the finite element method can be considered a very powerful numerical tool to analyze the stress distribution in structures with cutouts of complicated geometry. Some recent applications and references for determining the stress concentration using the finite element method can be found in [8, 27-42].

The current work calculates the stress concentration numerically by means of the developed finite element. Several numerical examples have been considered to affirm the accuracy of the present finite element in solving these kinds of problems. As examples, the stress concentration factors (SCFs) in perforated symmetric laminates with different fiber orientation angles are calculated. It is observed that the numerical findings are found to be in good agreement in comparison to those obtained using the analytical solution proposed by Arslan [25]. This affirms the accuracy and the performance of the proposed finite element formulation. This study was carried out to gain better understanding of the effect of the presence of notches on laminate strength. The failure theories used in this investigation are: Tsai-Wu (T-W) [43], Hashin-Rotem (H-R) [44] and Tsai-Hill (T-H) [26]. The strength values obtained using these theories are compared to each other in order to show the differences between them.

DESCRIPTION OF THE PROBLEM

The SCFs developed in the vicinity of cutouts in isotropic as well as in laminated plates, obtained numerically using the proposed finite element, were compared with the analytical findings calculated using the analytical approach developed by Arslan [25]. The numerical model used in this investigation is a square plate with a side equal to L = 100 cm. The plate is weakened by a circular cutout of radius R = 5 cm located in the center of the domain. Using these dimensions, the plate in this situation can be considered as an infinite plate and the numerically obtained results can be compared to those obtained analytically. The laminated plates are selected to be symmetric. Each layer in the laminate has a thickness equal to 0.1 cm. In this investigation, the elastic material properties that we used in this study are given in Table 1. E_1, E_2 and ν_{12} represent Young's modulus and Poisson's ratio in the principal fiber directions, and G_{12} represents the shear modulus.

NUMERICAL RESULTS AND DISCUSSION

Three numerical tests are proposed in the present investigation in order to affirm the accuracy and the efficiency of the present finite element. In the first numerical test, isotropic plates with circular notches are considered. The validity and the accuracy of the proposed finite element formulation are proved by comparing the results obtained numerically with those obtained using the exact analytical solution. The results of the convergence study are tabulated in Table 2. The second numerical example is considered to assess the capability of the present formulation for evaluating the stress distribution in symmetric laminated composite plates weakened by circular cutouts. In these tests, the circumferential stress values at any point α located on the edge of the notch are calculated. Laminated plates with different fiber orientation angles are used. The analytical and numerical findings are presented in Figures 5-10 and tabulated in Table 3. Finally, single layers with different ratios of E_1/E_2 are considered to understand the effect of the anisotropic ratio on the stress value. In these experiments, the stress distribution around a circular cutout is calculated using different fiber orientation angles.

In the first test, for which exact solutions are available, isotropic plates subjected to shear loading are considered. Since the strains and stresses are localized in the region near the cutout, a graded mesh that facilitates capturing high gradients is used. As shown in Figure 1, a very fine mesh, with smaller elements, is adopted in the region of stress location, while the remaining area, far from the notch region, is discretized with a coarse mesh to reduce the computational cost. This mesh refinement has been done in two stages. The first stage accomplishes refining the element size over the radial direction where the plate edge is divided into 4, 5, 6, 8, and 10 elements, respectively. The second stage accomplishes refining the areas where stress peaks occur. The cutout edge is divided into 10, 14, 20, 30 and 40 elements, respectively. For several mesh configurations, the maximum compressive SCF obtained using the present element and the exact solution obtained by Eq. (22) are tabulated in Table 2.

TABLE 1. **Material properties**

Materials	Elastic properties								
	E_1 [MPa]	E_2 [MPa]	G_{12} [MPa]	X [GPa]	Y [GPa]	X' [GPa]	Y' [GPa]	S' [GPa]	$\nu 12$
Isotropic	200	-	80	-	-	-	-	-	0.25
CF-T300	63.8	63.8	3.2	-	-	-	-	-	0.036
Woven glass-epoxy	15.8	15.8	2.8	-	-	-	-	-	0.25
Glass-epoxy	54.9	18.3	9.14	1055.5	28.1	1055.5	140.7	42.2	0.25
Graphite-epoxy	181	10.3	7.17	1500	40	1500	246	68	0.28

Remark: It should be noted that the cutout edge is divided also into 50 and 60 elements. However, the results were stable and there was no improvement in the results compared to 40 elements. Thus, 40 elements are adopted in the region of stress localization to reduce the computational cost.

Remark: To facilitate the comparison with the analytical results, uniform shear stress loading of 1 MPa is applied at the edge of the plate. At the edges where the plates were subjected to loading, each node "*i*" between two elements takes a load equal to the sum of the halves of the loads applied on the adjacent elements. For example, node *i* supports a load S = (S1+S2)/2 × 1 MPa. (see Fig. 1)

It is well shown from the convergence study (Table 2 and Fig. 2) that the proposed finite element presents a very good convergence speed towards the analytical solution, which indicates that the proposed finite element can accurately calculate the SCF in an isotropic plate under shear loading and the accuracy of the element is verified.

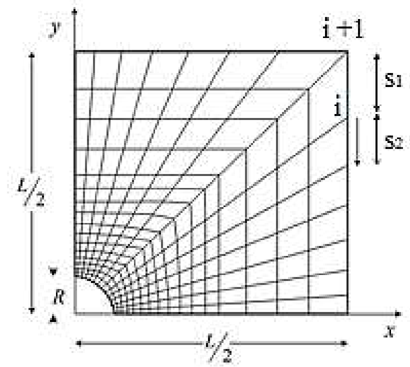

Fig. 1. Finite elements mesh of quarter of plate

TABLE 2. **Results of convergence study**

Materials	Present element					Arslan [25]
	(4×10)	(5×14)	(6×20)	(8×30)	(10×40)	
isotropic	–3.169	–3.311	–3.611	–3.904	–3.969	–4.000

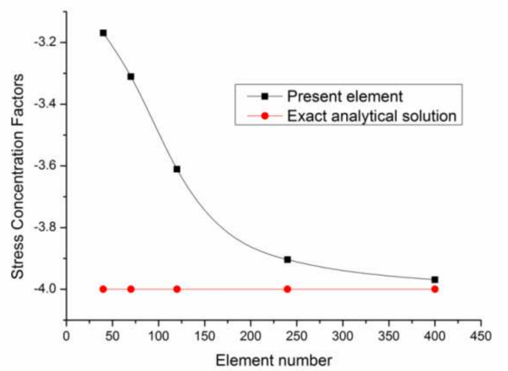

Fig. 2. Mesh refinement results

One can see in Figure 2 that by increasing the number of elements in both radial and circumferential directions, the stress concentration increases until it reaches the analytical value when the number of elements is (10×40). This mesh configuration is adopted in the remaining study as it provides satisfactory results.

After validating the performance of the proposed element, the second numerical test is considered to assess the capability of the proposed element to determine the stress in laminated composite plates weakened by a circular cutout. In these experiments, typical analytical and numerical results are presented to show the effect of the composite material properties, shear loading, fiber orientation, and anisotropic ratio on the stress concentration values.

Fiber orientation effect

The fiber orientation angle has an essential effect on the stress concentration and its location. To understand the effect of the fiber orientation angles on the stress distribution around notches, the stress function given by Eq. (22) and its parameters are coded using FORTRAN to illustrate the distribution of the circumferential stress. Isotropic and three different composite materials are considered in this investigation. The dimensions of the solution domain and the boundary loading condition are similar to the square plate used in the first test. In the present example, two graphite-epoxy and CF/T300 composite laminated plates are studied. The laminated plates are chosen to be symmetric cross-ply [0/90]s. In the second example, an eight-layered woven glass-epoxy plate is considered. The stacking sequence of the present symmetric laminated plate is chosen to be [0/30/60/90]s. In addition to the first two examples, a symmetric eight layered glass-epoxy plate is also considered. In this example, the stacking sequence of the proposed laminate is [0/15/30/45]s. Different types of materials are used in this investigation to see the effect of the material properties on the stress values. In the present investigation, the maximum SCFs are calculated for each layer in the laminate separately because the developed stresses in all the layers will not be equal due to the effect of the fiber orientation angle.

The stress distributions calculated in each particular layer depending on the circumferential location are presented graphically in Figures 3-5. Circumferential angle α ranges between 0° and 180°. The stress distributions are obtained using the analytical stress function proposed by Arslan [25]. For various composite materials, the maximum stress concentration values obtained using the proposed finite element and the analytical findings given by Eq. (22) are tabulated in Table 3. Regarding the CF-T300 laminated composite, it is noted that the variation in the stress concentration values depending on the circumferential location is exactly the same for both the top $\theta = 0°$ and bottom $\theta = 90°$ layers. Also, when Figure 3 is examined, it is observed that the maximum tensile and compressive SCF in absolute value is 3.692 for 0° and 90° fiber orientation angles. The stress concentration curves are smooth, and there is no sudden change in their slopes. In the graphite/epoxy laminate, the situation is quite different. The maximum values of stress concentrations are more significant than the stress concentrations in the CF-T300 laminate and the slope of the curves changes abruptly.

TABLE 3. **Stress concentration factors for various laminates under shear load**

		Arslan [25]		Present element	
Laminated plates	**Fiber orientation angle**	**Max. tensile SCF**	**Max. compressive SCF**	**Max. tensile SCF**	**Max. compressive SCF**
Isotropic	-	4.000	–4.000	3.969	–3.969
Graphite - epoxy	0°	6.498	–6.498	6.980	–5.120
	90°	6.498	–6.498	6.033	–4.398
Woven glass - epoxy	0°	3.496	–3.496	3.458	–3.194
	30°	4.482	–4.482	4.281	–3.661
	60°	4.482	–4.482	4.258	–3.667
	90°	3.496	–3.496	3.477	–3.214
Glass - epoxy	0°	4.184	–4.184	4.046	–3.846
	15°	4.933	–3.571	4.856	–3.337
	30°	5.512	–3.342	5.292	–3.074
	45°	5.727	–3.306	5.453	–3.002
CF/T300	0°	3.692	–3.692	3.934	–3.163
	90°	3.692	–3.692	3.937	–3.165

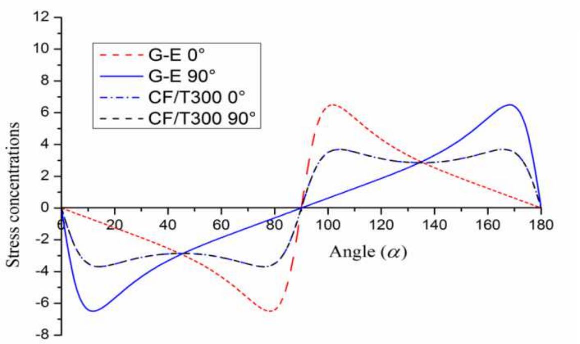

Fig. 3. Stress concentrations σ_α / S versus α for two laminate composites (graphite-epoxy and CF/T300)

In this case, the top and bottom layers have the same maximum stress concentration value that equals 6.498, but unlike the previous case, the critical circumferential angle, i.e. the location for which the maximum stress concentration occurs, is not the same. Concerning the woven glass-epoxy composite laminate, it is noted that the variation of the stress concentration values depending on the circumferential location is exactly the same for both the top $\theta = 0°$ and bottom $\theta = 90°$ layers.

Moreover, when Figure 4 is examined, it is observed that the maximum tensile and compressive SCF in absolute value is 3.496. The stress concentration curves are smooth and there is no sudden change in their slopes for both the top and bottom layers. For layers that have an orientation angle equal to $\theta = 30°$ and $\theta = 60°$, the situation is quite different. The maximum values of stress concentrations are higher than in both fiber orientation angles $\theta = 0°$ and $\theta = 90°$, and the slope of the curves changes abruptly. In this case, these layers have the same maximum stress concentration value, which equals 4.482. Still, unlike the previous fiber orientation angles 0° and 90°, the critical circumferential angle is not the same.

From Figure 5, it is noted that the maximum stress concentration values are around 5.6 for both fiber orientation angles $\theta = 30°$ and $\theta = 45°$. The critical circumferential angle is around $\theta = 135°$.

When the figures are examined altogether, the trend of the curves changes regarding the circumferential angle. These figures indicate that as the fiber orientation angle is increased, the critical region where the stress values are the maximum moves to the end of the horizontal axis.

For both the CF-T300 and woven glass-epoxy plates, if one wants to explain why the stress variation is exactly the same for $\theta = 0°$ and $\theta = 90°$, it is because these two materials have the same modulus of elasticity in the principal and transverse fiber directions. Also, one can immediately note that for two fiber orientation angles symmetrical with respect to $\alpha = 45°$, the tensile stress takes the same maximum stress concentration values in two locations symmetrical with respect to $\alpha = 135°$.

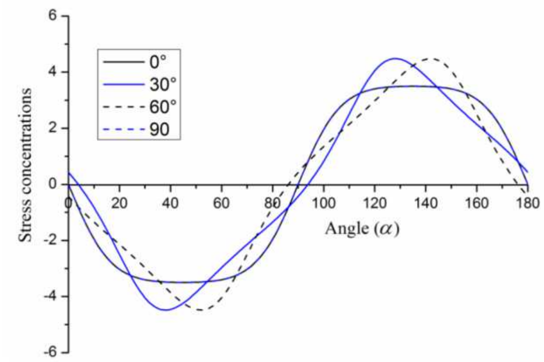

Fig. 4. Stress concentrations σ_α / S versus α for four layered woven glass-epoxy composite

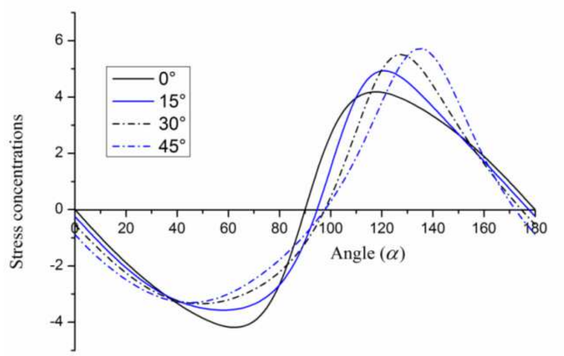

Fig. 5. Stress concentrations σ_α / S versus α for four layered glass/epoxy composite

For different material composite plates, the maximum stress concentration value is 3.692 when CF-T300 (E_1/E_2 = 1) was used; it is 4.482 when woven glass-epoxy (E_1/E_2 = 1) was used, while it is 5.727 when glass-epoxy (E_1/E_2 = 3) was used and it is 6.498 when graphite-epoxy (E_1/E_2 = 18) was used (see Table 3). Firstly, one can directly note that as the (E_1/E_2) ratio increases, i.e. as the load carrying capacity of the fibers increases, the stress concentration value also increases because of more loads in the fibers. Secondly, it is well observed in Table 3 that the values of the stress concentrations, when the CF-T300 material is considered, are found to be the smallest compared to the stresses obtained for the remaining materials under consideration. As a result of this investigation, the stress concentration is dependent on the anisotropic ratio. On the other hand, one can see that the stress values obtained by the present finite element are found to be in good agreement compared to the analytical findings.

For a better understanding of the effect of fiber orientation on the maximum stress values, an in-depth study is proposed using woven glass-epoxy and glass--epoxy plates with differently oriented fibers. These layered plates are subjected to shear loading. The analytical results are compared to the numerical findings obtained using the proposed finite element.

The maximum values of the SCF developed in the vicinity of the cutout corresponding to different fiber orientation angles ranging between 0° and 90° with 5° increments are presented in Figures 6 and 7. In both these figures, one can see that the maximum values of the tensile and compressive stresses, in both materials under consideration, are affected considerably by changing the fiber orientation angle and the obtained results demonstrate its effect. It is again noted that the maximum values of stress concentration are a function of the fiber orientation angle and each layer has its proper stress concentration. For single layered plates subjected to shear loading conditions, it is observed that by increasing the fiber orientation angle, both the tensile and absolute compressive stresses increase until they reach their maximum values for $\theta = 45°$, then they decrease to reach their minimum values for $\theta = 90°$.

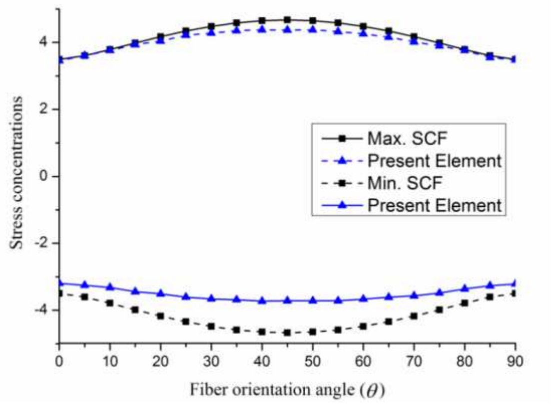

Fig. 6. Stress concentration factors σ_α / S versus fiber orientation angles for woven glass-epoxy plates

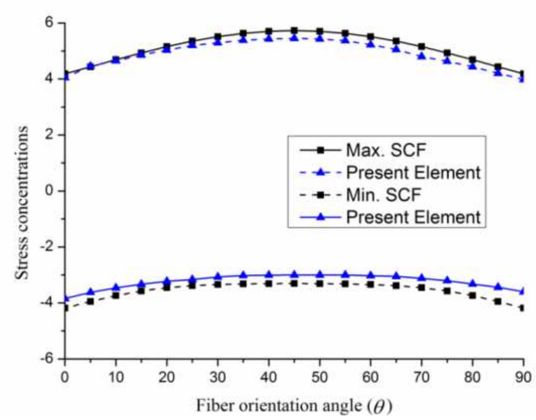

Fig. 7. Stress concentration factors σ_α / S versus fiber orientation angles for glass-epoxy plates

On the other hand, for glass-epoxy plates the compressive stress concentration decreases with the increase in fiber orientation angle to reach the minimum negative value at $\theta = 45°$ then it increases. One can immediately note that the SCFs exhibit symmetric distribution with respect to $\theta = 45°$. Moreover, one can also see good agreement between the results given by the analytic solution and those obtained by the present element.

Effect of degree of orthotropy

As a result of our previous tests, the maximum stress concentration value and the position where it occurs vary as the fiber orientation changes. Additionally, the stress concentration values also change depending on the ratio of the elasticity modulus of the fiber direction to the transverse one E_1/E_2. Our effort now will be focused on presenting a parametric study, which aims at understanding the effect of anisotropic ratio E_1/E_2 on the values and positions of stress concentration around notches in orthotropic composite plates. To show the effect of this parameter on the stress distribution, three different orientation angles are considered. Considering these cases, the stress distributions are presented graphically for $\theta = 0°$ (see Fig. 8a), $\theta = 90°$ (see Fig. 8b), and $\theta = 45°$ (see Fig. 8c) with different E_1/E_2 ratios. The material proprieties are taken as E_2 = 1, G_{12} = 0.5 and ν_{12} = 0.25.

The effect of anisotropic ratio E_1/E_2 on the stress concentration value for various fiber orientations is presented graphically in Figure 8. Using the glass/epoxy material properties and by increasing the volumetric ratio of the fibers, higher values of E_1/E_2 can be obtained. The stress concentration is significantly affected by the increase in the anisotropic ratio for $\theta = 0°$ and 90°. It is observed that the stress concentration values increase by increasing the anisotropic ratio (Figs. 8 a,b) and ratio E_1/E_2 = 100 gives the highest stress concentration value. From these figures, it is noticed that the stress concentration curves exhibit a symmetric distribution. The stress concentration values are negative up to $\alpha = 90°$, after which the values are positive and entirely symmetric with respect to $\alpha = 90°$.

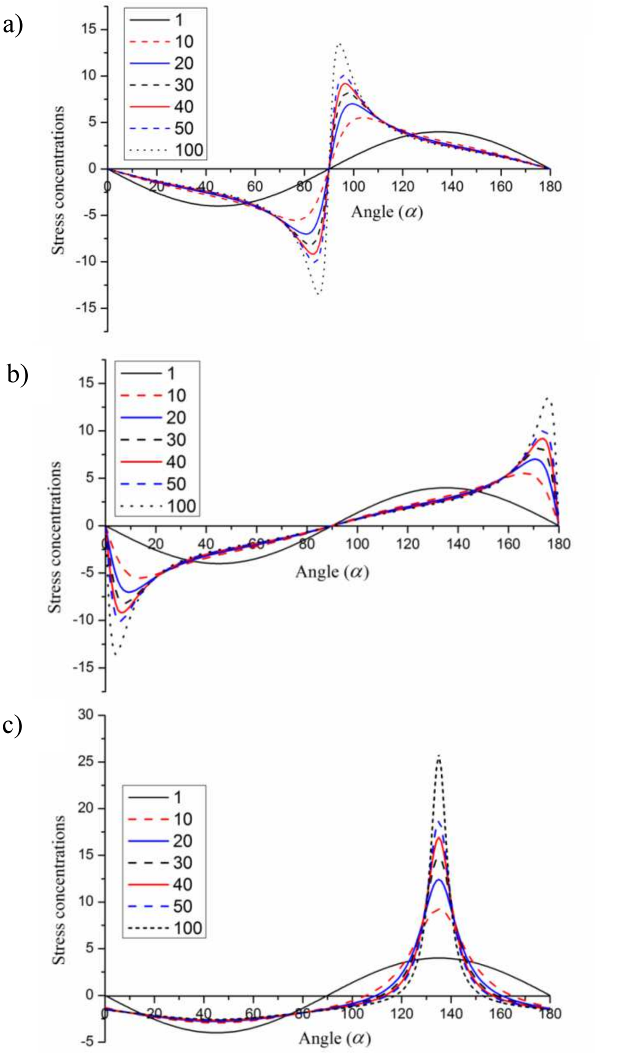

Fig. 8. Stress concentration σ_α / S versus α for different modulus ratio E_1 / E_2 and fiber orientation: a) $\theta = 0°$, b) $\theta = 90°$, c) $\theta = 45°$

In Figure 8a, the maximum stress concentration value is around 13.58. The locations of the maximum stress values of the negative and positive stresses are near to $\alpha = 90°$. In Figure 8b, the maximum stress concentration value is again 13.58 as in Figure 8a, but the position where the maximum occurs is dissimilar in the two cases. In the first case (Fig. 8a), the negative and positive values are in the region of $\alpha = 90°$ and in the second case (Fig. 8b), the location of the maximum negative value is around $\alpha = 5°$ and the location of the maximum positive value is around $\alpha = 175°$. In both Figures 8a and 8b, the stress distributions around the holes are not affected too much by the change in the E_1/E_2 ratio, except in regions where the maximum occurs. In areas far from the maximum stress concentration position, the curves have practically the same slope and the stress concentration values are quite close for all the E_1/E_2 ratios except the isotropic plate case. When Figure 8c is examined, symmetry cannot be observed as in Figures 8a and 8b. In this case where the fiber orientation is 45°, the stress concentration value increases up to 25.81, which is greater than when $\theta = 0°$ and $\theta = 90°$. It is obviously observed in Figure 8c that the increase in the E_1/E_2 ratio has no effect on the stress distribution around the cutouts except regions around $\alpha = 135°$, which needs to be strengthened when fibers are designed to have $\theta = 45°$.

Effect of fiber orientation on failure load

The main aim of the present section is to study the effect of the fiber orientation angle on the failure load. Different failure theories have been used, including Hashin-Rotem (H-R), Tsai-Hill (T-H) and Tsai-Wu (T-H), to calculate the strength of glass-epoxy plates subjected to shear loading conditions. Both glass-epoxy laminates with and without holes have been considered to see the effect of the presence of notches on the laminate strength. The failure load has been calculated and presented in Figure 9. Based on the FPF theory, the minimum strength value will be taken as the failure load. It is well known that one of the main characteristics of laminate plates is that each layer can have different fiber orientation angles.

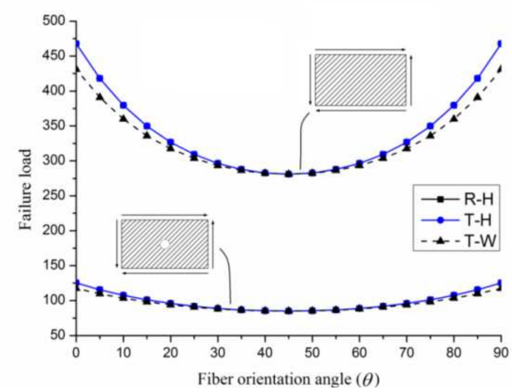

Fig. 9. Effect of notch presence and fiber orientation angles on failure load of glass-epoxy laminates

To ascertain its effect, different fiber orientation angles ranging from 0° to 90° with 5° increments are considered. The failure load for various fiber orientation angles is calculated and compared to each other hand, one can see which layer is the critical one and which fails first in a laminated plate made with different fiber orientations. The FPF load obtained for different layers subjected to shear loading are presented in Figure 9. In order to see the influence of the material properties, a similar investigation is carried out using graphite-epoxy laminates. It is well observed in the figures that the obtained results using different failure theories are almost the same and they have similar characteristics. One can see from Figures 9 and 10 that when the applied load is parallel or perpendicular to the fiber direction, the effect of the presence of circular cutouts on the strengths of the plate is found to be maximum. On the other hand, since graphite fibers are much stronger than the glass fibers, the strength of graphite-epoxy laminates is much higher than glass-epoxy laminates.

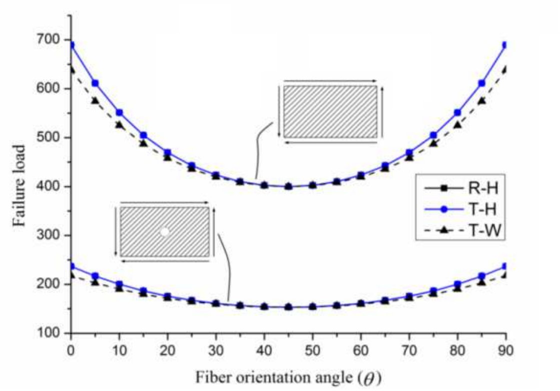

Fig. 10. Effect of notch presence and fiber orientation angles on failure load of graphite-epoxy laminates

As shown in Figures 9 and 10, by increasing the fiber orientation value, the failure load decreases until it reaches the minimum value at $\theta = 45°$, then it increases to reach the maximum value at $\theta = 90°$. As a last observation, the strength values for plates without holes are much higher than those obtained for plates with holes for both the materials under consideration.

CONCLUSIONS

The recently developed quadrilateral finite element was used to predict the stresses in cross and angle-ply laminates containing circular cutouts. Different plates with different material properties were considered in the present investigation to understand the behavior of perforated laminated plates subjected to shear loading conditions. The numerical results are compared to the analytical findings obtained using the Arslan [25] approach. By studying different examples, the results obtained by the proposed element are in good agreement with those obtained using the analytical solutions given by Arslan. As a result of the present study the stress concentration and failure load are highly dependent on the fiber orientation angle for single plies under external shear loading. The presence of a circular cutout considerably decreases the strength of the plates compared to unnotched plates. The strength degradation will be at the minimum when the fiber orientation angles are equal to $\theta = 0°$ and $\theta = 90°$.

REFERENCES

[1] Topal U., Uzman Ü., Frequency optimization of laminated composite angle-ply plates with circular hole. Materials & Design 2008, 29(8), 1512-1517.

[2] Bouzgou A.A., Khechai A., Tati A., Stress concentration and deflection in isotropic and orthotropic plates with opening. Finite element study, Revue des Composites et des Matériaux Avancés 2015, 25.

[3] Khechai A. et al., Finite element analysis of stress concentrations in isotropic and composite plates with elliptical holes, In: Design and Modeling of Mechanical Systems-II, Springer 2015, 427-436.

[4] Bouzgou A.A., Tati A., Khechai A., Analysis of deflection in isotropic and orthotropic rectangular plates with central opening under transverse static loading, In: Applied Mechanics, Behavior of Materials, and Engineering Systems, Springer 2017, 399-409.

[5] Khechai A. et al., Strength improvement and stress analysis of e-glass laminated plates with circular notches using digital image correlation, In: Composite and Nanocomposite Materials – From Knowledge to Industrial Applications, IntechOpen 2019.

[6] Khechai A., Mohite P., Optimum design of perforated symmetric laminates using evolutionary algorithm, Journal of Composite Materials 2019, 53(23), 3281-3305.

[7] Khechai A. et al., A general solution for stress resultants around a circular cutout in laminate plates under different in-plane loadings: analytical and experimental investigations, Archive of Applied Mechanics 2018, 88(7), 1187--1208.

[8] Khechai A., Tati A., Guettala A., Numerical study of the effect of presence of geometric singularities on the mechanical behavior of laminated plates, Journal of the Institution of Engineers (India), Series C 2018, 99(6), 717-728.

[9] Ukadgaonker V., Rao D., A general solution for stress resultants and moments around holes in unsymmetric laminates, Composite Structures 2000, 49(1), 27-39.

[10] Ukadgaonker V., Kakhandki V., Stress analysis for an orthotropic plate with an irregular shaped hole for different in-plane loading conditions, Part 1, Composite Structures 2005, 70(3), 255-274.

[11] Sharma D.S., Stress concentration around circular/elliptical/triangular cutouts in infinite composite plate, Proceedings of the World Congress on Engineering, 2011.

[12] Pilkey W.D., Pilkey D.F., Bi Z., Peterson's Stress Concentration Factors, John Wiley & Sons 2020.

[13] Kaltakci M.Y., Stress concentrations and failure criteria in anisotropic plates with circular holes subjected to tension or compression, Computers & Structures 1996, 61(1), 67-78.

[14] Lekhnitskii S.G., Anisotropic plates, Gordon and Breach, 1968.

[15] Savin G.N., Stress Concentration Around Holes, Pergamon Press, 1961.

[16] Muskhelishvili N.I., Some basic problems of the mathematical theory of elasticity, Noordhoff Groningen 1953, 15.

[17] Xu X., Sun L., Fan X., Stress concentration of finite composite laminates with elliptical hole, Computers & Structures 1995, 57(1), 29-34.

[18] Tsukrov I., Novak J., Effective elastic properties of solids with defects of irregular shapes, International Journal of Solids and Structures 2002, 39(6), 1539-1555.

[19] Vigdergauz S., Optimal stiffening of holes under equibiaxial tension. International Journal of Solids and Structures 1993, 30(4), 569-577.

[20] Exadaktylos G., Liolios P., Stavropoulou M., A semi-analytical elastic stress-displacement solution for notched circular openings in rocks, International Journal of Solids and Structures 2003, 40(5), 1165-1187.

[21] Cherkaev A. et al., The cavity of the optimal shape under the shear stresses, International Journal of Solids and Structures 1998, 35(33), 4391-4410.

[22] Dharmin P., Khushbu P., Chetan J., A review on stress analysis of an infinite plate with cut-outs, International Journal of Scientific and Research Publications 2012, 2(11), 1-7.

[23] Nagpal S., Jain N., Sanyal S., Stress concentration and its mitigation techniques in flat plate with singularities – A critical review, Engineering Journal 2012, 16(1), 1-16.

[24] Green A.E., Zerna W., Theoretical elasticity, Courier Corporation 1992.

[25] Arslan H.M., Stress concentrations in fiber reinforced composite plates containing circular holes subjected to shear stress, Kuwait Journal of Science and Engineering 2007, 34(1B), 1.

[26] Azzi V., Tsai S.W., Anisotropic strength of composites, Experimental Mechanics 1965, 5(9), 283-288.

[27] Kalita K., Halder S., Static analysis of transversely loaded isotropic and orthotropic plates with central cutout, Journal of the Institution of Engineers (India), Series C 2014, 95(4), 347-358.

[28] Nagpal S., Jain N.K., Sanyal S., Three dimensional parametric analyses of stress concentration factor and its mitigation in isotropic and orthotropic plate with central circular hole under axial in-plane loading, Journal of the Institution of Engineers (India), Series C 2016, 97(1), 85-92.

[29] Joshi S. et al., SCF in isotopic & orthotropic plates, Part II: Biaxial Load, Materials Today: Proceedings 2017, 4(2), 2639-2644.

[30] Jadvani N. et al., SCF in Isotopic & Orthotropic Plates, Part I: Uniaxial load, Materials Today: Proceedings, 2017, 4(2), 2632-2638.

[31] Jadvani N. et al., Non-dimensional stress analysis of orthotropic laminates, Materials Focus 2017, 6(1), 63-71.

[32] Kumar A. et al., Analysis of stress concentration in orthotropic laminates, Procedia Technology 2016, 23, 156-162.

[33] Kalita K., Shinde D., Thomas T.T., Non-dimensional stress analysis of an orthotropic plate, Materials Today: Proceedings 2015, 2(4-5), 3527-3533.

[34] Khechai A. et al., Strength degradation and stress analysis of composite plates with circular, square and rectangular notches using digital image correlation, Composite Structures 2018, 185, 699-715.

[35] Barski M., Static and fatigue strength of composite plates with holes, Composites Theory and Practice 2014, 1, 3-7.

[36] Kwiatkowski D., Nabialek J., The study of crack resistance of the pp composites with talc on the basis of the stress intensity factor, Composites Theory and Practice 2009, 9(4), 369-372.

[37] Hufenbach W. et al., Multistable fibre-reinforced composites with cutouts, Kompozyty (Composites) 2003, 3, 7.

[38] Hufenbach W. et al., Enhanced strength models for notched laminates with finite outer boundaries, Kompozyty (Composites) 2003, 3, 7.

[39] Hufenbach W. et al., Analiza koncentracji naprężeń dla drewna wzmocnionego materiałem tekstylnym, Kompozyty (Composites) 2002, 2, 5.

[40] Chwał M. et al., Residual stresses in multilayered composites – General overview, Composites Theory and Practice 2016, 16(3), 132-138.

[41] Bondyra A., Miarka S., Pastuszak P.D., Static compression of GFRP plate with hole-3D scanning comparative evaluation, Composites Theory and Practice 2016, 16(4), 218-222.

[42] Barski M., Kędziora P., Chwał M., Design of rectangular composite plates with circular holes, Composites Theory and Practice 2016, 1(16), 52-57.

[43] Tsai S.W., Hahn H.T., Introduction to Composite Materials, Routledge 2018.

[44] Hashin's failure criteria for unidirectional fiber composites, India Institute of Technology, Kanpur, 1980.

