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MATHEMATICAL MODEL OF MODIFIED CLASSICAL LAMINATION THEORY FOR FOAM CORE SANDWICH COMPOSITES

Elastic properties are important quantities in the modelling and analysis of sandwich composite structures. The stability of sandwich composites mainly depends on their elastic properties, which in turn depend on the elastic properties of its constituents namely, the core and face skin. Several models have been proposed to predict the elastic constants of core materials such as honeycomb and foam. A foam core may be open-cell foam or closed-cell foam. The present work is focused on the hexagonal cells of a honeycomb grid core and closed-cell polymer syntactic foam core. The honeycomb is considered to be orthotropic with nine independent elastic properties. However, the overall structural performance of the honeycomb core is mainly influenced only by out-of-plane elastic properties. On the other hand, the syntactic foam is considered to beisotropic with two independent elastic constants namely, the modulus of elasticity and Poisson's ratio. The face skin material may be isotropic with two independent elastic constants or orthotropic with nine elastic constants under three-dimensional loading. The present work is focused on predicting the elastic properties of a honeycomb core, syntactic foam and a glass/epoxy composite using existing theoretical models. Thereafter, the elastic properties of the syntactic foam and glass/epoxy composite are later used to establish the elastic constants for syntactic foam core sandwich composites using modified classical lamination theory (MCLT). The results reveal that the reviewed theoretical models for the honeycomb core, syntactic foam, fiber--reinforced polymeric (FRP) glass/epoxy face skins and sandwich composites are validated by the experimental results.

Keywords: **honeycomb grid, syntactic foam, FRP, sandwich composite**

INTRODUCTION

Elastic properties are important quantities in the modelling and analysis of sandwich structures. The stability of sandwich composites mainly depends on its elastic properties, which in turn depend on the elastic properties of its constituents namely, the core and face skin. Several models have been proposed to predict the elastic constants of core materials such as honeycomb and syntactic foam. The honeycomb is considered to be orthotropic with nine independent elastic properties. Nevertheless, the overall structural performance of the honeycomb core is mainly influenced only by out-of-plane elastic properties. On the other hand, the syntactic foam is considered to beisotropic with two independent elastic constants namely, the modulus of elasticity and Poisson's ratio. The face skin material may be isotropic (aluminum, titanium, copper-nickel etc.) with two independent elastic constants or orthotropic (fiber--reinforced polymer composites) with nine elastic constants under three-dimensional loading. In the present work, it is worthwhile highlighting suitable theoretical approaches like Gibson and Ashby's approach for predicting the elastic properties of a hexagonal honeycomb core grid structure, Porfiri and Gupta's approach for evaluating the elastic properties of hollow microspheres embedded in syntactic foam and the mechanics of material approach for assessing the elastic properties of fiber-reinforced polymer (FRP) composites. The present work gives a review of these theoretical models, and focuses on including these approaches to build a mathematical model of modified classical lamination theory regarding prediction of the elastic properties of syntactic foam core sandwich composites and their experimental validation.

GIBSON AND ASHBY'S APPROACH FOR HONEYCOMB CORE

Gibson and Ashby [1] consider the honeycomb core shown in Figure 1 to bean orthotropic material with nine elastic constants namely, Young's moduli E_{xx}, E_{yy}, E_{zz} in the X, Y and Z directions, respectively, shear moduli G_{xy}, G_{yz}, G_{zx} in the XY, YZ and ZX planes, respectively, and Poisson's ratios υ_{xy}, υ_{yz}, υ_{zx} in the XY, YZ and ZX planes, respectively. Nonetheless, Schwingshackl et al. [2] reported that for a honeycomb core, the elastic properties that govern structural sturdiness are E_{zz}, G_{xz} and G_{yz} (out-of-plane properties). Hence, these properties were evaluated using the Gibson and Ashby model.

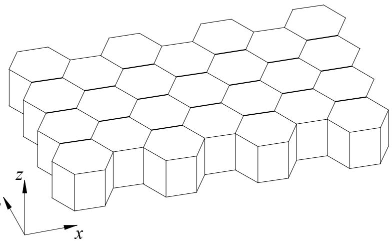

Fig. 1. Typical hexagonal honeycomb core

Figure 2 shows a typical honeycomb cell structure. The density of the honeycomb core depends on its geometry and the density of its material. Similarly, the elastic property of a honeycomb core is a function of its geometry and the elastic properties of the material used to fabricate the honeycomb core.

a) b)

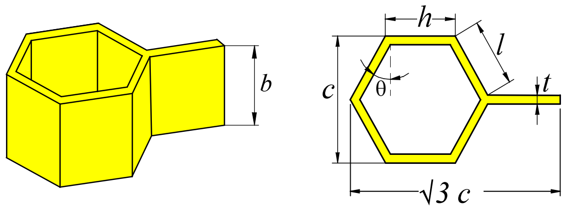

Fig. 2. Typical honeycomb cell structure: a) honeycomb cell, b) geometry of honeycomb cell

In Figure 2b, l and h are the length and edges of the hexagon, c ($=l\sqrt{3}$) is the cell size, b is the height of the honeycomb core (or thickness of the honeycomb core), t is the thickness of the face of the honeycomb and θ is the semi-angle between two faces of the honeycomb. It should be noted that for a regular hexagon, $h = l$, $\theta = 30°$. The geometric details, density (ρ_{paper}) and elastic properties (E_{paper} and G_{paper}) of the honeycomb material i.e. Nomex® paper of cell size 20 mm (N20) and kraft paper of cell sizes 20 mm (K20), 15 mm (K15), 10 mm (K10) and 5mm(K5) are presented in Table 1.

TABLE 1. **Geometric details, density and elastic properties of honeycomb material**

Parameters	Nomex paper	Kraft paper			
	N20	K20	K15	K10	K5
h [mm]	11.5	11.5	8.6	5.7	2.8
l [mm]	11.5	11.5	8.6	5.7	2.8
ρ_{paper} [kg/m^3]	724	690			
E_{paper} [GPa]	3.4	3			
G_{paper} [GPa]	2.38	2.1			
θ, degrees	30°				
T [mm]	0.13				
B [mm]	12.5				

According to Gibson and Ashby, the density and out-of-plane compression modulus (E_{zz}) can be determined from Equations (1) and (2), respectively. The out-of-plane shear moduli, G_{xz} and G_{yz} can be determined by Equations (3) and (4), respectively. The calculated values of these properties are presented in Table 2.

$$\rho_{RIPH} = \left(\frac{t}{l}\right)\left\{\frac{\left(1+\frac{h}{l}\right)}{\left(\frac{h}{l}+Sin\theta\right)Cos\theta}\right\}\rho_{Paper} \tag{1}$$

$$E_{zz} = \left(\frac{t}{l}\right)\left\{\frac{\left(1+\frac{h}{l}\right)}{\left(\frac{h}{l}+Sin\theta\right)Cos\theta}\right\}E_{paper} \tag{2}$$

$$G_{yz} = \left(\frac{t}{l}\right)\left\{\frac{Cos\theta}{\left(\frac{h}{l}+Sin\theta\right)}\right\}G_{paper} \tag{3}$$

$$G_{xz} = G_{xz\_lower} + \frac{0.787}{\left(b/l\right)}\left(G_{xz\_upper} - G_{xz\_lower}\right) \tag{4}$$

where G_{xz_upper} and G_{xz_lower} are the upper boundary and lower boundary shear moduli in the XZ plane of the honeycomb core and are obtained from Equations (5) and (6), respectively.

$$G_{xz\_upper} = \left(\frac{t}{l}\right)\left\{\frac{\left(\frac{h}{l}+Sin^2\theta\right)}{\left(\frac{h}{l}+Sin\theta\right)Cos\theta}\right\}G_{paper} \tag{5}$$

$$G_{xz\_lower} = \left(\frac{t}{l}\right)\left\{\frac{\left(\frac{h}{l}+Sin\theta\right)}{\left(1+\frac{h}{l}\right)Cos\theta}\right\}G_{paper} \tag{6}$$

TABLE 2. **Theoretically predicted properties of resin impregnated paper honeycomb core**

Samples	Density ρ [kg/m^3] (Eq. (1))	Compression modulus E_{zz} [MPa] (Eq. (2))	Shear modulus $G_{yz\_}$ [MPa] (Eq. (3))	Shear modulus G_{xz} [MPa] (Eq. (4))
N20	12.54	58.93	36.92	15.47
K20	11.96	52	32.57	13.65
K15	15.94	69.33	39.40	18.20
K10	23.92	104	53.05	27.30
K5	47.84	208	94	54.60

The deviations between the theoretically predicted and experimentally determined values of the density and compression modulus as found by Amith Kumar SJ et al. [3] are given in Table 3. There may be deviation between the experimental and theoretical results because of the variation in the hexagonal shapes of the honeycomb cells in the core and non-uniform coating of resin during impregnation. There is no ASTM standard prescribed for the experimental evaluation of out-of-plane shear modulus.

TABLE 3. **Deviation between theoretical and experimental values of density and compressive modulus for honeycomb core grid structure**

Samples	Deviation in density ρ [%]	Deviation in compression modulus E_{zz} [%]
N20	4.3	15.6
K20	17	2.5
K15	12.9	9
K10	0.3	8.3
K5	12.8	7.1

PORFIRI AND GUPTA'S APPROACH FOR SYNTACTIC FOAM

The elastic properties of fiber-reinforced polymeric composites have been predicted using various approaches such as the mechanics of material approach [4], numerical approach [5, 6], self-consistent field approach [7, 8], variational approach [9, 10], semi-empirical approach [11] and the differential scheme [12-16]. Out of these approaches, differential approaches were considered for the development of theoretical models for particulate composites. In the present investigation, Porfiri and Gupta's [13, 17] coupled nonlinear differential Equations (7) and (8) were used for the theoretical prediction of the Young's modulus and Poisson's ratio of syntactic foams (a particulate composite), as this model considers the effect of wall thickness of the hollow microspheres (dry fly ash cenospheres) present in the foam.

$$\frac{dE}{dV_{ceno}} = f_E(E_{ceno}, \upsilon_{ceno}, E, \upsilon, \eta)\frac{E}{\left(1-\frac{V_{ceno}}{V_{\text{ceno(max)}}}\right)} \tag{7}$$

$$\frac{d\upsilon}{dV_{ceno}} = f_\upsilon(E_{ceno}, \upsilon_{ceno}, E, \upsilon, \eta)\frac{\upsilon}{\left(1-\frac{V_{ceno}}{V_{\text{ceno(max)}}}\right)} \tag{8}$$

where f_E and f_υ are real functions defined for the Young's modulus and Poisson's ratio of the syntactic foam [13, 17], E_{ceno} and υ_{ceno} are the Young's modulus and Poisson's ratio of the dry fly ash cenospheres, respectively. V_{ceno} and $V_{ceno(max)}$ are the volume fraction of the dry fly ash cenospheres and the maximum possible volume fraction (packing limit) of cenospheres in the syntactic foam composite, respectively; η is the radius ratio of the cenospheres (the ratio of the inner to outer radius). Equations (7) and (8) can be integrated to determine the Young's modulus and Poisson's ratio of syntactic foam core composites using the initial conditions $E = E_m$ and $\upsilon = \upsilon_m$ as $V_{ceno} = 0$, where E_m and υ_m are the Young's modulus and Poisson's ratio of the matrix (phenolic resin) material, respectively. Extensive experimentation has been conducted to determine the maximum packing limit (maximum possible volume fraction) of dry fly ash cenospheres in the phenolic resin matrix. It was observed that the maximum packing factor of cenospheres solely depends on their size. For monosized cenospheres, a packing factor up to 60% for random loose packing and body centered cubic packing, 63.7% for random close packing, 52.36% for simple cubic packing, 74.05% for face centered cubic packing and hexagonal cubic packing can be achieved. However, in the present investigation, cenospheres of different sizes (a mean diameter 150 microns) were used as the filler for the syntactic foam for which a maximum packing factor of 90% was achieved. It was observed that the threshold cenosphere volume fraction in syntactic foam is 72%, beyond which the binding ability of the phenolic resin matrix decreases. Table 4 presents the properties of the dry fly ash cenospheres, phenolic resin and their proportions considered for evaluating the elastic properties of syntactic foam.

TABLE 4. **Properties of syntactic foam constituents and their values**

Component	Property	Symbol	Unit	Value
Dry fly ash cenospheres	Young's modulus	E_{ceno}	GPa	17
	Poisson's ratio	υ_{ceno}	---	0.21
	Density	ρ_{ceno}	kg/m^3	450
	Volume fraction	V_{ceno}	---	0.7272
	Maximum packing factor	$V_{ceno(max)}$	---	0.9
	Radius ratio	η	---	0.9
Phenolic resin matrix	Young's modulus	E_m	GPa	2.51
	Poisson's ratio	υ_m	---	0.35
	Density	ρ_m	kg/m^3	1200
	Volume fraction	V_m	---	0.2727

The theoretically predicted Young's modulus (E_c) and Poisson's ratio (υ_c) of the syntactic foam composite were found to be 1.99 GPa and 0.25 GPa, respectively. Since syntactic foam is considered an isotropic material, its shear modulus was evaluated by means of Equation (9) and was found to be 0.796 GPa. The density (ρ_c) of the syntactic foam was determined by the simple rule of mixtures using Equation (10) and was found to be 654.54 kg/m^3. These values agree well with experimental values reported by Amith Kumar S J et al. [3].

$$G_c = \frac{E_c}{2(1+\upsilon_c)} \tag{9}$$

$$\rho_c = \frac{\rho_{ceno}\rho_m}{\rho_{ceno}M_m + \rho_m M_{ceno}} \tag{10}$$

where ρ_{ceno}, ρ_m, M_{ceno}, M_m are the density of the dry fly ash cenospheres, density of the matrix (phenolic resin), mass fraction of the dry fly ash cenospheres and mass fraction of the matrix (phenolic resin), respectively.

MECHANICS OF MATERIAL APPROACH FOR ORTHOTROPIC FACE SKINS

By using rule of mixture relationships from the mechanics of materials approach, it is possible to predict the elastic properties of the *E*-glass/epoxy composite. For a unidirectional *E*-glass/epoxy composite, the longitudinal Young's modulus (E_l), transverse Young's modulus (E_t), and Poisson's ratio (υ_{lt}) were calculated using Equations (11), (12) and (13), respectively.

$$E_l = E_f V_f + E_m V_m \tag{11}$$

$$E_t = \frac{E_f E_m}{E_f V_m + E_m V_f} \tag{12}$$

$$\upsilon_{lt} = \upsilon_f V_f + \upsilon_m V_m \tag{13}$$

where E_f and E_m are the Young's modulus of the fiber (*E*-glass fiber) and the matrix (epoxy), respectively; υ_f and υ_m are the Poisson's ratios of the *E*-glass fiber and epoxy, respectively; V_f and V_m are the volume fraction of the *E*-glass fiber and epoxy, respectively.

The in-plane shear modulus (G_{lt}) was determined by the rule of mixtures using Equation (14) and the Halpin--Tsai semi-empirical relation using Equation (15). The interlaminar shear modulus (G_{lz}) was also evaluated by the Tsai and Hahn semi-empirical stress--partitioning parameter using Equation (16).

$$G_{lt} = \frac{G_f G_m}{G_f V_m + G_m V_f} \tag{14}$$

$$G_{lt} = G_m \frac{(1+V_f)G_f + V_m G_m}{V_m G_f + (1+V_f)G_m} \tag{15}$$

$$G_{lz} = G_m \frac{V_f + n_{lz}(1-V_f)}{n_{lz}(1-V_f) + V_f\left(\frac{G_m}{G_f}\right)} \tag{16}$$

where $n_{lz} = \dfrac{3-4\upsilon_m + \left(\frac{G_m}{G_f}\right)}{4(1-\upsilon_m)}$

From the values of the elastic properties determined for unidirectional plies of the *E*-glass/epoxy composite and also by incorporating factor K defined in Equation (17), it is possible to predict the Young's moduli (E_{fs}) of the 0°/90° woven *E*-glass fabric/epoxy composite face skin in the warp (E_{11}) and weft (E_{22}) directions using Equations (18) and (19), respectively, and Poisson's ratio (υ_{12}) using Equation (20) [18].

$$K = \frac{N_1}{N_1 + N_2} \tag{17}$$

where N_1 and N_2 are the numbers of yarns in the warp and weft direction. Since *E*-glass fabric is balanced, the numbers of yarns in the warp and weft directions are equal (i.e. $K = 0.5$).

$$E_{fs} = E_{11} = KE_l + (1-K)E_t \tag{18}$$

$$E_{fs} = E_{22} = (1-K)E_l + KE_t \tag{19}$$

$$\upsilon_{12} = \frac{\upsilon_{lt}}{K + (1-K)\left(\frac{E_l}{E_t}\right)} \tag{20}$$

It can be noted that for woven fabric, $\upsilon_{12} = \upsilon_{21}$. The density, volume fractions and elastic properties of the constituent materials of the *E*-glass/epoxy composite are displayed in Table 5. The elastic properties of the *E*-glass/epoxy composite face skin as predicted by the aforementioned approach are compared with the experimental values in Table 6.

TABLE 5. **Properties of constituent materials of *E*-glass/epoxy composites**

Component	Property	Symbol	Unit	Value
***E*-glass fiber**	Young's modulus	E_f	GPa	72.4
	Poisson's ratio	υ_f	---	0.22
	Shear modulus	G_f	GPa	29.6
	Density	ρ_f	kg/m^3	2500
	Volume fraction	V_f	---	0.324
Epoxy resin matrix	Young's modulus	E_m	GPa	2.75
	Poison's ratio	υ_m	---	0.38
	Shear modulus	G_m	GPa	0.99
	Density	ρ_m	kg/m^3	1200
	Volume fraction	V_m	---	0.676

TABLE 6. **Elasticity properties of *E*-glass/epoxy face skin**

Property	Symbol	Unit	Theoretical	Experimental	% Deviation
Longitudinal Young's modulus	E_{11}	GPa	14.655	14.39	1.8
Transverse Young's modulus	E_{22}	GPa	14.655	14.39	1.8
In-plane Poisson's ratio	$\upsilon_{12} = \upsilon_{21}$	---	0.088	0.086	2.3
In-plane shear modulus	G_{12}	GPa	1.862	1.83	1.7
Interlaminar shear modulus	$G_{13} = G_{23}$	GPa	1.766	----	----

MODIFIED CLASSICAL LAMINATION THEORY FOR SANDWICH COMPOSITE

Depending on the geometry (aspect ratio, core thickness), constituent material properties and the type of loading, a sandwich composite panel can be analyzed using classical lamination theory by including the transverse shear effects [19]. Figure 3 shows a typical sandwich composite panel. This sandwich panel can be treated as a three-ply laminate consisting of a core and two face skins.

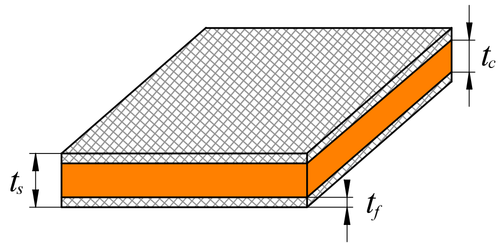

Fig. 3. Typical sandwich composite

If the two face sheets are identical in thickness and properties, the laminate is symmetric, that is, B_{ij} = 0. The in-plane extensional sandwich panel stiffness was determined by means of Equation (21).

$$A_{ij} = 2\left[Q_{ij}\right]_f t_f + \left[Q_{ij}\right]_c t_c \tag{21}$$

where $\left[Q_{ij}\right]_f$ is the reduced stiffness matrix of the *E*-glass/epoxy composite face skins and was determined using Equation (22).

$$\left.\begin{aligned} [Q_{11}]_f &= \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}} \\ [Q_{22}]_f &= \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}} \\ [Q_{12}]_f &= \frac{\upsilon_{12}E_{22}}{1-\upsilon_{12}\upsilon_{21}} \\ [Q_{66}]_f &= G_{12} \end{aligned}\right\} \tag{22}$$

For an isotropic core material, the elements of reduced stiffness matrix $\left[Q_{ij}\right]_c$ are determined using Equation (23).

$$\left.\begin{aligned} [Q_{11}]_c = [Q_{22}]_c &= \frac{E_c}{1-\upsilon_c^2} \\ [Q_{12}]_c = [Q_{21}]_c &= \frac{\upsilon_c E_c}{1-\upsilon_c^2} \\ [Q_{66}]_c = G_c &= \frac{E_c}{2(1+\upsilon_c)} \end{aligned}\right\} \tag{23}$$

Furthermore, the transverse shear stiffness [A_{44}], [A_{55}] for sandwich composites with an orthotropic *E*-glass/epoxy face skin and isotropic syntactic foam core were determined by means of Equation (24).

$$\left.\begin{aligned} A_{44} &= 2\left(G_{23}\right)_f t_f + G_c t_c \\ A_{55} &= 2\left(G_{13}\right)_f t_f + G_c t_c \end{aligned}\right\} \tag{24}$$

where E_{11}, E_{22}, E_{12}, υ_{12}, υ_{21}, G_{12}, G_{13} and G_{23} are the elastic properties of the woven *E*-glass/epoxy composite laminate. E_c, υ_c and G_c are the elastic properties of the syntactic foam core. The elastic properties of sandwich composite (SF) with a core of syntactic foam and face skins of *E*-glass/epoxy composite laminate as a function of laminate extensional stiffness can be determined using Equations (25)-(31). The theoretically predicted and experimentally evaluated values of the elastic properties for sandwich composite (SF) are compared in Table 7, along with the percentage deviation.

Longitudinal Young's modulus

$$\left(E_x\right)_s = \frac{1}{t_s}\left[A_{11} - \frac{A_{12}^2}{A_{22}}\right] \tag{25}$$

Transverse Young's modulus

$$\left(E_y\right)_s = \frac{1}{t_s}\left[A_{22} - \frac{A_{12}^2}{A_{11}}\right] \tag{26}$$

Major Poisson's ratio

$$\left(\upsilon_{xy}\right)_s = \frac{A_{12}}{A_{22}} \tag{27}$$

Minor Poisson's ratio

$$\left(\upsilon_{yx}\right)_s = \frac{A_{12}}{A_{11}} \tag{28}$$

In-plane shear modulus:

$$\left(G_{xy}\right)_s = \frac{A_{66}}{t_s} \tag{29}$$

Transverse shear modulus:

$$\left(G_{yz}\right)_s = \frac{A_{44}}{t_s} \tag{30}$$

Transverse shear modulus:

$$\left(G_{xz}\right)_s = \frac{A_{55}}{t_s} \tag{31}$$

where A_{11}, A_{22}, A_{12} and A_{66} are the elements of the extensional stiffness matrix A_{ij}.

TABLE 7. **Elastic properties of sandwich composite (SF)**

Elastic properties	Symbol	Unit	Theoretical	Experimental	% Deviation
Longitudinal Young's modulus	$(E_x)_s$	GPa	4.469	4.38	1.9
Transverse Young's modulus	$(E_y)_s$	GPa	4.469	4.38	1.9
Major Poisson's ratio	$(\upsilon_{xy})_s$	---	0.148	----	---
Minor Poisson's ratio	$(\upsilon_{yx})_s$	---	0.148	----	---
In-plane shear modulus	$(G_{xy})_s$	GPa	1.0023	0.961	4.1
Transverse shear modulus	$(G_{yz})_s$	GPa	0.9838	1.035	5.2
Transverse shear modulus	$(G_{xz})_s$	GPa	0.9838	1.035	5.2

CONCLUSIONS

The density and elastic properties of the constituent materials required for the construction of sandwich composites and the sandwich composite as a whole were theoretically predicted using well-accepted theoretical models. Based on the results, the following important conclusions were drawn,

1. The theoretically predicted out-of-plane(flatwise) compressive modulus (E_{zz}) and density (ρ_{RIPH}) of a resin impregnated paper honeycomb (RIPH) core structure using Gibson and Ashby's approach were found to be in good agreement with the experimentally determined values.
2. The theoretically predicted tensile modulus and Poisson's ratio of syntactic foam using Porfiri and Gupta's differential scheme approach were found to be in good agreement with the experimentally determined values.
3. The in-plane elastic properties of woven *E*-glass/ epoxy composite face skins obtained utilizing the mechanics of materials approach (rule of mixtures) conform with the experimental values.
4. The elastic properties of sandwich composites (SF) determined with the modified classical lamination theory agree well with those obtained by experimentation.

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