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## A NEW APPROACH TO DETERMINING THE ELASTIC MODULUS OF STRUCTURAL MATERIALS

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Stiffness characteristics are often decisive in the choice of material for structural parts. At the same time, the process of their determination for anisotropic materials does not fully satisfy the requirements in terms of reliability and reproducibility. This work is devoted to the development of an approach for determining the moduli of elasticity. An analysis of the Timoshenko approaches developed to estimate the shear component of deflection in transverse bending is presented. The drawbacks preventing their use as a basis for modern methods of determining the elastic components of structural materials are noted. An approach for determining the elastic moduli is proposed, the basis of which are the maximum values of deflections and angles of rotation of the cross-sections of a specimen under three-point transverse bending. The relationship between angular and linear displacements under the considered type of loading is established, which allows stable and reliable values of elasticity moduli to be obtained from the data of angular displacements. The acceptability of the proposed approach for determining the elastic moduli of both isotropic materials and composites is shown.

**Keywords:** composite, modulus of elasticity, bending, experimental determination, deflections

### INTRODUCTION

The development of new composite materials and the study of their properties contribute to the growth of unique features of these materials compared to traditional ones. This factor leads to the rapid expansion of their use in various branches of science and technology. They have become especially attractive in aerospace and shipbuilding industries, which impose high requirements to the reliability assessment of structural elements made of them. This is primarily determined by the accuracy and reliability of determining certain properties of these materials. Thus, for structural parts made of composite materials, the most important of these properties are the elastic moduli. They are

among the main characteristics necessary to assess the possibility of using structural materials in various industries. Therefore, the methods of determining the noted characteristics should not cause any doubt in the reliability, stability or reproducibility of their values. However, the real situation in this matter contradicts this condition.

Firstly, the developed analytical methods based on the elastic properties of the reinforcing fibres, their arrangement and the elastic properties of the matrix give a wide range of elastic modulus values. The Voigt method shows the highest values and the Reiss method the lowest. Detailed analyses of these methods are presented in [1-4].

Some of them are rather cumbersome and inconvenient for practical implementation.

Therefore, their experimental evaluation does not yet meet the requirements. Despite the fact that many years have passed since the advent of composite materials, to date the main most reliable experimental method for determining the moduli of elasticity is the tensile test method for isotropic materials. The method is standardised [5-7]. These standards are used for all types of composite materials, regardless of their reinforcement structure, without sufficient justification or evidence of their acceptability. The main disadvantages of the noted standards are the general approach to the determination of elastic moduli of both isotropic and composite materials, as well as the high requirements for their manufacture and relatively labour-intensive process of testing. The process of bending specimen testing is therefore receiving increasing attention.

## STATE OF THE ART AND PROBLEM STATEMENT

The issues related to determining the elasticity moduli of composite materials under bending are considered in a number of works. The main attention in them is paid to evaluation of the influence of various factors on the accuracy of deflection es-

timations and its components, as well as to the estimation of real values of composite material elastic moduli [8-11]. Therefore, some of the standards for the determination of elastic moduli from three-point bending experiments have been developed with these factors in mind, e.g. [12-14]. The main requirement for their use is strict control over the accuracy of specimen manufacture, the established value of the ratio of specimen length to specimen thickness,  $l/h$ , and consideration of the established values of relative strains. The determined value of the apparent modulus of elasticity in bending,  $E_b$ , depends significantly on the ratio  $l/h$ , especially when  $l/h \leq 25$  [15]. Nevertheless, the relationship between  $E_b$  and the real modulus of elasticity of the material,  $E$ , at different values of  $l/h$  has not yet been established by anyone. The possibility of determining the real value of  $E$  (based on test data) at small values of  $l/h$  is currently not available at all. The values of the moduli of elasticity for two types of structural materials, calculated using the usual equation [9]:

$$E_b = \frac{P(l/h)^3}{4by} \quad (1)$$

are presented in Table 1. Hereinafter:  $P$  is the force applied to the specimen in the centre of the span;  $b$  is the width of the specimen,  $y$  is the deflection of the specimen in the middle of the span.

TABLE 1. Dependence of modulus of elasticity in three-point bending of structural materials on change in  $l/h$  and  $P$

Steel					Boron plastic				
$l/h$	$P$ , KN	$y^e$ , mm	$E_b$ , GPa	$E_b/E^+$	$l/h$	$P$ , KN	$y^e$ , mm	$E_b$ , GPa	$E_b/E^+$
40	0.300	1.334	211.659	1.001	50	0.072	0.731	150.799	1.010
30	0.430	0.778	219.304	1.036	30	0.206	0.550	123.860	0.830
20	1.276	0.782	191.847	0.907	25	0.308	0.519	114.447	0.767
10	3.000	0.247	178.503	0.844	15	0.274	0.186	60.890	0.408

Note:  $y^e$  is the deflection measured experimentally on the bottom surface of the specimen under the point of load application; tensile modulus of elasticity for steel  $E^+ = 211.520$  GPa; for boron plastic  $E^+ = 149.176$  GPa.

The  $y$  values were determined on the linear part of the  $P$ - $y$  dependence, as it is stipulated by the current standards. The data given in the table show that bending tests of both isotropic and composite materials allow reliable values of the modulus of elasticity to be obtained only at a set value

$l/h$ , not less than 40. Reducing this value leads to a difference between the determined elastic modulus values and those obtained from tensile tests. This makes this method inefficient because of the impossibility to use the results obtained at  $l/h < 40$ .

It is obvious that the modernization of standards and tightening of requirements for sample preparation have not brought about any significant progress in achieving the set goal. As before, assessment of the deflection shift is carried out using the Timoshenko equation:

$$\frac{1}{E_b} = \frac{1}{E_x} + \frac{1,2}{G} \left( \frac{h}{l} \right)^2 \quad (2)$$

It is currently employed in many works as a basis for studying the components of maximum deflection and its shear component. For this purpose, a term is added to this dependence, taking into account the necessary factor. An example can be found in works [16, 17]. In them, the same term is added to the main dependence, taking into account local deformation effects. Let us consider the role of the additional term in solving the problem using the example of [16]. The hypothesis adopted in this case is clearly shown in the figure in [16]. Its essence is that when loading a sample for three-point bending, the maximum deflection under the point of applying the load consists of individual components (from bending, shear, and local deformations). The latter consist of components from supports and from the applied load. These two components have constant values that are distributed throughout the span. The components of deflection from bending and from shear have the same curve character with a minimum value at the supports and a maximum under the point of application of the load. The numerical values of these deflection components are not presented in the work. The ratios of local deformation effects to deflection from bending, depending on parameter  $l/h$ , are presented in the figure for three types of CMs. These data show that the maximum value of the local component with respect to the bending deflection component for unidirectional and woven CFRP, at  $l/h \geq 15$ , does not exceed 2.5% and 1.0%, respectively, while for woven FRP, at  $l/h \geq 15$ , it is less than 1%. The shear component of deflection is also presented there, which, relative to the component from bending at  $l/h \geq 15$  for unidirectional and woven CFRP, respectively, does not exceed 15% and 9%, and less than 3% for

FRP. The ratio of  $E_b$  to  $E$  for all three materials under consideration is above 90%. Analysis of the presented data, without taking into account the errors introduced by the accepted hypothesis, suggests that the modernization of the Timoshenko equation does not have a noticeable effect on the value of maximum deflection during bending. The data in works [16, 17] do not agree with the assessment of the distribution of local deformations in isotropic [18, 19] and anisotropic [2, 20] materials. In the latter, it is clearly shown that, despite the relatively high values of local deformations, they quickly fade and do not have a noticeable effect on the value of the determined characteristic. Therefore, the aim of this work is to assess the capabilities of the Timoshenko approaches to establishing the components of deflection during three-point bending and to develop approaches based on the received data to obtain comparable values of elastic moduli for any values of  $l/h$ .

## TIMOSHENKO APPROACHES

There are three approaches developed by Timoshenko for assessing shear deformations in the three-point bending of isotropic beams. They have different theoretical initial bases for creation, but practically the same final results. The specimen loading scheme used in these approaches [18] is shown in Fig. 1.

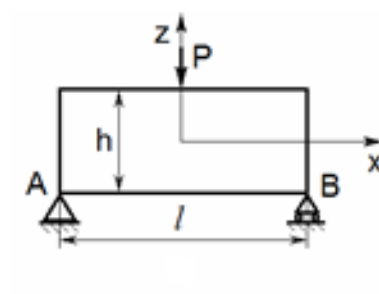


Fig. 1. Specimen loading scheme for estimating shear deformations in bending

The first approach is obtained from the solution of a plane problem using integer polynomials, where the stress function is represented as an integer polynomial of the fifth degree [18].

An equation is given for calculating the beam deflection, taking into account the shear and local stresses from concentrated force  $P$ :

$$y = \frac{P(l/h)^3}{4Eb} + 0.74 \frac{3}{2} \frac{Pl}{4hbg} \quad (3)$$

The first part is a standard equation for calculating beam deflection without taking shear into account, and the second is an addition to the deflection from shear deformations and local stresses. From the second part of Eq. (3) it follows that the shear component of the deflection is equal to the relative shear angle multiplied by the span length, i.e.

$$y^s = \frac{1.11P}{4Gbh} l \quad (4)$$

here the shear stress is:

$$\tau = \frac{1.11P}{4bh} \quad (5)$$

The latter is 2.7 times higher than the former, and the value of the interlayer shear modulus of steel calculated by one of the methods developed on the basis of Timoshenko's approaches occurred to be many times lower than its real value.

$$\tau_{max} = \frac{3P}{4bh} \quad (6)$$

Thus, two factors – a high value of total deflection and low values of shear stresses, clearly indicate one of the possible reasons preventing achievement of the set goal, namely, incorrect determination of the components of deflection during bending. This follows from the analysis carried out by the author of the approaches under consideration. In his work [18], it is noted: “The obtained stress distribution completely coincides with that given by the elementary theory of bending”. It is also noted there that the real picture of stress distribution has some differences from the accepted one. This is due to the fact that local stresses are added to the bending stresses at the point of applying concentrated force  $P$ , which quickly fade, and do not have a noticeable effect on the entire picture of the stress distribution. Local stresses, including normal tangential stresses, quickly fade with distance from the point of applying the force. Moreover, already at a distance

equal to the height of the beam, as a rule, they can be ignored [19].

The second approach of Timoshenko is based on the use of the differential equation of the curved axis of the beam [21]. According to this approach, the total deflection of a rectangular beam, under three-point bending by force  $P$  in the middle of span  $l$ , is equal to:

$$y = \frac{Pl^3}{48EJ} \left( 1 + 3.90 \frac{h^2}{l^2} \right) \quad (7)$$

Here, as in the first approach, the component of the deflection from shear is added in the middle of the span to the deflection from bending. This does not agree with the experimental results, where the value of the total deflection with a decrease in  $l/h$  turns out to be significantly lower than with the initial span. Even an increase in  $P$  by several times does not lead to the equality of these deflections. This indicates that the additive is part of the total deflection.

The third approach is described in [19]. It is based on establishing the curvature of the deflection curve taking into account the nature and character of the stress distribution for the loading scheme considered here. It is shown in [19] that for steel samples, the obtained dependencies for the total deflection in all approaches are completely identical. Therefore, its further consideration does not seem appropriate.

The analysis shows that the considered basic approaches to assessing the maximum deflection and its components during bending have been extremely poorly studied, and their acceptability for determining shear deformations even of isotropic materials has not been proven in any way.

## EXPERIMENTAL STUDIES AND THEIR RESULTS

The experimental study of the maximum deflection components in bending tests was first carried out on steel specimens whose characteristics have been well studied, and there are proven and reliable methods for determining its elastic and shear properties. The experiments were carried out by means of tension and three-point bending of

prismatic beams with a rectangular cross-section. The tensile tests were carried out to establish the exact values of the shear modulus and elastic modulus for the purpose of comparing them with the values obtained from three-point bending. In this case, the shear moduli were determined only for an indirect assessment of the correctness of determining the shear component of the deflection. The tensile and bending tests were conducted on an MTS machine in accordance with the current standards noted above. The deflection was measured on the lower surface of the beam in its middle, exactly under the point of application of the load. The measurement was performed using a calibrated steel plate, at the end of the fastening zone of which two foil strain gauges were glued, connected to a computer. The deflection was measured automatically from the beginning to the end of specimen loading. The calculated values of the characteristics, calculated according to the first approach, using Eq. 3 for different values of the

$l/h$  ratio, are collected in Table 2. The  $y^s$  values are calculated taking into account the value of the shear modulus of steel, obtained by another reliable method [22, 23]. It is equal to 82.170 GPa. The modulus of elasticity of steel in bending ( $l/h = 40$ ) was  $E = 211.659$  GPa; in tension  $E^+ = 211.520$ . The dimensions of the sample are  $b$ , mm /  $h$ , mm = 17.0 / 5.0.

Analysis of the data collected in Table 2 shows that the use of Eq. 3 is acceptable for assessing the shear component of the deflection of steel samples. The estimated deflection value, calculated without taking into account the shear (the first part of Eq. 3), has only a slight (0.20%) excess of its experimental value  $y^e$  at  $l/h = 40$ . The shear component with a change in two parameters – an increase in  $P$  and a decrease in  $l/h$ , as can be seen from Table 2, increases in relation to its value for large spans (see  $y^s / y_l^s$ ). In this case, the value of the total deflection decreases, and no changes in the  $P - y$  dependence are observed.

TABLE 2. Characteristics of steel specimens calculated using Eq. 3 and initial data used for their calculation

$l$ , mm	$l/h$	$P$ , KN	$P / P_l$	$y^e$ , mm	$y^*$ , mm	$y^s$ , mm	$y^s / y_l^s$	$y$ , mm	$y / y^e$	$G$ , GPa
200	40	0.300	1.000	1.334	1.3368	0.0024	1.000	1.339	1.004	–
150	30	0.430	1.433	0.778	0.8080	0.0026	1.074	0.811	1.042	–
100	20	1.276	4.253	0.782	0.7110	0.0052	2.127	0.716	0.916	6.434
50	10	3.000	10.000	0.247	0.2090	0.0061	2.502	0.215	0.870	13.137

Note:  $P_l$ ,  $y_l^s$  are the values for the largest span;  $y^*$  is the calculated deflection without taking into account the shear.

The values of the characteristics calculated according to the second approach (Eq. 7) for steel specimens are given in Table 3. All the characteristics are calculated using the same data that were previously used in the first approach. Compared to

the first, the second approach yields larger values of the shear component of the deflection, but the calculated values of deflection  $y^*$  turned out to be identical to those obtained in the first approach for all the values of  $l/h$ .

TABLE 3. Characteristics of steel specimens calculated using Eq. 7 and initial data used for their calculation

$l$ , mm	$l/h$	$P$ , KN	$P / P_l$	$y^e$ , mm	$y^*$ , mm	$y^s$ , mm	$y^s / y_l^s$	$y$ , mm	$y / y^e$	$G$ , GPa
200	40	0.300	1.000	1,3340	1.3368	0.0033	1.000	1.3401	1.005	–
150	30	0.430	1.433	0,7780	0.8080	0.0035	1.071	0.8115	1.043	–
100	20	1.276	4.253	0,7820	0.7110	0.0069	2.126	0.7180	0.918	8.966
50	10	3.000	10.000	0,2470	0.2090	0.0082	2.503	0.2172	0.879	22.637

The use of the second approach leads to results similar to those obtained using the first one. This is especially demonstrated by the ratios of deflections  $y^s / y_l^s$  and  $y / y^e$  of both approaches. The presented data show that the calculated shear component of the deflection for all the studied ratios  $l/h$  has a very small value compared to the deflection measured under the point of application of the load. Thus, at  $l/h = 40$ , the component of the deflection from shear is only 0.18% of  $y^e$ , and 2.35% of  $y^e$  at  $l/h = 10$ . A decrease in  $l/h$  leads to an increase in  $y^s$ , similar to the data calculated using Eq. 7 (see Table 3). The change in  $y^s$  is influenced not only by parameter  $l/h$ , but also by applied force  $P$ . The calculated values of the shear modulus are also greatly underestimated, as are those presented in Tables 2 and 3.

It should be noted that the approaches under consideration are not suitable for assessing the shear modulus of isotropic materials, since at  $l/h \geq 30$  the given Eqs. 3, and 7 do not allow its values to be calculated due to the excess of the calculated value of the total deflection  $y$  compared to  $y^e$ . At  $l/h \leq 20$  the determined values of the shear modulus turn out to be very underestimated (see Tables 2, 3) compared to the real ones ( $G = 82.170$  GPa).

The analysis of the studies shows that all the approaches considered were developed to evaluate the effect of shear deformation on deflection only for isotropic materials whose shear modulus is in exact and excellent agreement with the modulus of elasticity. For anisotropic materials, including composite materials, no such consistency has been yet established. Therefore, in this case, the application of these approaches to the noted materials is not possible. One of the main shortcomings of the considered approaches – the summation of shear and transverse bending deflections – should be mentioned again.

The determined values of the modulus of elasticity, even for isotropic materials, are significantly underestimated [10, 24]. This is also due to the fact that the determined value of the modulus of elasticity in three-point bending is identified by determining the maximum deflection based on classical Eq. 1. This excludes from consideration the fact that the modulus of elasticity is a material

characteristic and, unlike  $y$ , does not depend on parameter  $l/h$  or other external factors that do not disturb its integrity and structure. Its exact value, even for isotropic materials, is determined from tensile experiments [18].

To take into account the influence of shear deformations on deflection in three-point bending, while keeping the value of the elastic modulus of the material constant, we consider the dependences for determining the deflection and the angle of rotation in any cross-section of the specimen [10]:

$$y_x = \frac{Px^3}{12EJ} - \frac{Pl^2}{16EJ}x \quad (8)$$

$$\Theta_x = \frac{Px^2}{4EJ} - \frac{Pl^2}{16EJ} \quad (9)$$

Here  $x$  is the distance from the left support of the beam;  $\Theta_x$  is the angle of rotation of the cross-section at distance  $x$  from the left support. The maximum value of deflection occurs at  $x = l/2$ , i.e. under the point of applying force  $P$ , hence, in this case,

$$y = y_{max} = \frac{Pl(l/h)^2}{4FE} \quad (10)$$

Here  $F$  is the cross-sectional area of the specimen. The maximum angle of rotation occurs at the support, i.e. at  $x = 0$ . Hence

$$\Theta = \frac{3P(l/h)^2}{4EF} \quad (11)$$

and at  $x = l/2$ , i.e. directly under the point of applying force  $P$ ,  $\Theta = 0$ . Hereinafter  $\Theta = \Theta_{max}$  is the maximum angle of rotation of the specimen cross-section. From Eqs. 10 and 11 it follows that

$$y = y_{\Theta} = \frac{1}{3}l\Theta \quad (12)$$

or

$$\Theta = \frac{3y^e}{l} \quad (13)$$

Here  $y^e$  is the deflection measured at  $l/h = 40$ ;  $y_{\Theta}$  is the linear displacement caused by the rotation angles of the specimen sections.

Availability of the obtained exact values of angular and linear displacements at two characteristic points makes it possible to visualise more clearly the whole process of their change in the

process of loading. For this purpose, angular displacements are transformed into linear displacements according to dependence (12). Their values are plotted at characteristic points – at one of the supports and under the point of applying force  $P$ . Linear displacements also take place at these points. Taking into account the linear character of the  $P - y$  dependence for all the selected values of  $l/h$ , as well as the linear character of change of angular displacements (angles of rotation of the specimen sections) on the sections from the supports to the point of force application [19], the two  $y_\Theta$  points and the corresponding two  $y_{max}$  points can be connected by a straight line. The scheme of displacement variation is shown in Fig. 2. It follows from the above that the angular and linear displacements are interrelated. The maximum deflection under the point of force application does not contain a shear component. Therefore, the use of the Timoshenko approaches, taking into account the ‘build-up’ of the maximum deflection, is not confirmed either analytically or experimentally. The considered approaches enable reliable values to be determined of the shear component in bending of isotropic materials included in the total deflection, but not the addition to the deflection. The latter is their drawback. The latter introduces noticeable changes in the methods of determining their moduli of elasticity as well. This is especially true for monotropic CMs.

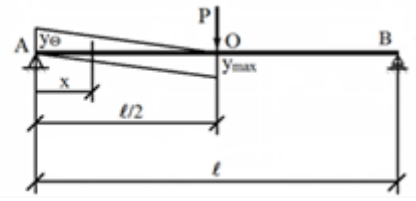


Fig. 2. Diagram of displacement variation in beam section under consideration

The estimation of their moduli of elasticity in the direction of reinforcement by the existing methods developed for isotropic materials gives values that are overestimated by about two times. This is confirmed, for example, by [25], where it is shown that the experimentally determined bending stiffness of a composite beam is, as a rule, less than that calculated from the value of the elastic modulus determined in tension. Therefore, the interrelation of the individual bending and tensile data can be taken into account to exclude such facts. This relationship is used in the presented approach to determine the modulus of elasticity in bending. The modulus of elasticity in this case can also be determined on the basis of angular displacement data converted to linear displacements according to Eq. 12, as follows:

$$E = \frac{P(l/h)^3}{4by_\Theta} \quad (14)$$

Experimental data on the estimation of the modulus of elasticity of steel by linear and angular displacements are contained in Table 4.

TABLE 4. Values of elastic modulus of steel under three-point bending with different  $l/h$ , calculated using Eq. 14 and initial data used in the calculation

$l/h$	$P$ , KN	$P^x$ , KN	$\Theta$ , rad	$y_\Theta$ , mm	$E$ , GPa	$E/E^+$
40	0.300	0.300	0.0200	1.333	211.817	1.001
30	0.430	0.711	0.0161	0.805	212.093	1.003
20	1.276	2.400	0.0213	0.710	211.433	1.000
10	3.000	19.200	0.0125	0.208	212.104	1.003

Note:  $h = 5.0$  mm;  $b = 17.0$  mm;  $E^+ = 211.520$  GPa;  $E = 211.659$  GPa;  $y_e = 1.334$  mm.

When calculating linear displacements, the following condition was used:  $P(l/h)^3 = \text{const.} = 0.300 \text{ KN} \times 403 = 19200 \text{ KN}$ . It should be noted that the measured deflection is the same for all  $l/h$ . The modulus of elasticity, determined by  $y_e$ , is

also the same for all  $l/h$ . The obtained values are given below the table. The table shows the results of the angular displacement calculation. The values of  $\Theta$  were calculated by Eq. 11,  $y_\Theta$  by Eq. 12,

and the values of  $E$  given in the table were determined by Eq. 14.

As can be seen from Table 4, the values of the moduli of elasticity obtained by angular and linear displacements, for all ratios of  $l/h$  are very close and agree well with the modulus of elasticity in tension, that is, they represent the real values of the modulus of elasticity of the material. The validity of the obtained values for angular displacements is well confirmed by equality  $y_{\Theta} = y$  at

$l/h = 40$  and the total value of  $y = 1.333$  calculated by Eq.14. Similar data were obtained for composite materials.

The experimental and calculated data of the modulus of elasticity in the bending of orthogonal-reinforced (1:1) fiberglass made on the basis of veneer are given in Table 5. It shows that at all  $l/h$  ratios the values of elastic moduli are in good agreement.

TABLE 5. Values of elastic modulus of 1:1 fiberglass under three-point bending with different  $l/h$ , calculated using Eq. 14 and initial data used in the calculation

$l/h$	$P$ , KN	$\Theta$ , rad	$y_{\Theta}$ , mm	$E$ , GPa	$E/E^+$
19.61	0.102	0.0098	0.655	22.920	1.002
9.804	0.458	0.0110	0.367	22.955	1.000
4.902	0.856	0.0052	0.086	22.939	0.999

Note:  $h = 10.2$  mm;  $b = 12.8$  mm;  $E^+ = 22.941$  GPa;  $E = 22.500$  GPa;  $y_e = 0.656$  mm.

The experimental and calculated bending elastic modulus data of boron plastic (1:0) are shown in Table 6. These data show good agreement be-

tween the elastic modulus values obtained at different  $P$  and  $l/h$ . Thus, it is possible to obtain stable and reliable values of the modulus of elasticity at values  $l/h < 40$ .

TABLE 6. Values of elastic modulus of boron plastic 1:0 under three-point bending with different  $l/h$ , calculated using Eq. 14 and initial data used in the calculation

$l/h$	$P$ , KN	$\Theta$ , rad	$y_{\Theta}$ , mm	$E$ , GPa	$E/E^+$
50	0.072	0.0220	0.731	150.817	1.011
30	0.206	0.0230	0.452	150.840	1.011
25	0.308	0.0230	0.391	150.742	1.010
15	0.274	0.0075	0.075	150.808	1.011

Note:  $h = 2$  Mm;  $b = 20.4$  mm;  $E^+ = 149.176$  GPa;  $y_e = 0.731$  mm.

The proposed approach makes it possible to determine the values of elastic moduli on specimens with different  $l/h$  ratios simply and efficiently enough for structural materials of various types, based on the determined deflection value. The measured deflection under the point of force application  $P$  can serve as a test for the reliability of the determined characteristic since the calculated value of the maximum deflection obtained by Eq. 12 excludes the possible influence of other factors on its value.

The problem is quite important in many areas of engineering (including coatings of various

kinds, e.g. [26]). Thus, such analysis should be developed further in future studies on this subject.

## CONCLUSIONS

1. A simple approach for determining the elastic moduli of structural materials has been proposed, the basis of which is the maximum values of deflections and angles of rotation of the cross-sections of the specimen under three-point bending.
2. The relationship between the maximum values of deflections and rotation angles of the



specimen sections, which are mutually related to each other during loading and inseparable, has been established.

3. It has been established that the maximum deflection under the point of load application does not contain a shear component of deformations, which excludes the possibility of using the Timoshenko approaches with the presence of a shear additive for the development of methods for determining the elasticity and shear moduli of structural materials.

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