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COMPOSITES THEORY AND PRACTICE

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OPTIMUM WEIGHT DESIGN OF MULTILAYERED FIBER COMPOSITE PLATES WITH DISCRETE PLY ANGLES AND UNCERTAINTY IN PLY THICKNESSES

The presented paper discusses the minimum weight design of multilayered fiber composite plates with tolerances in individual ply thicknesses. These tolerances are given by the maximum acceptable deviation of every individual ply thickness from its nominal value. The robustness of the design is achieved by diminishing the design state variable (buckling load factor) by the product of arbitrary assumed tolerances and appropriate sensitivities. The proposed approach is illustrated with examples of a simply supported rectangular laminated plate design under uni- and bi-axial compression. The minimum weight identified by the total number of layers is found to assure plate stability. For the discussed analysis, buckling load sensitivity formulas with respect to ply thicknesses are given. Based on these relations, the impact of the discussed variations on the optimal laminate stacking sequence and buckling mode shape is studied in detail. The achieved results emphasize the importance of robust design opposed to merely nominal approaches.

Keywords: **laminate plate, optimization, manufacturing tolerances, robust design, structural stability**

OPTYMALIZACJA WIELOWARSTWOWYCH PŁYT KOMPOZYTOWYCH Z WŁÓKNAMI UKIERUNKOWANYMI DYSKRETNIE I NIEDOKŁADNOŚCIAMI GRUBOŚCI LAMIN

Przedstawiono zagadnienie optymalizacji wielowarstwowych płyt kompozytowych z włóknami ukierunkowanymi dyskretnie z uwzględnieniem występowania tolerancji grubości poszczególnych lamin. Rozważane tolerancje zostały zdefiniowane jako maksymalne dopuszczalne odchylenia rzeczywistej grubości każdej z warstw od jej wartości nominalnej. W zaproponowanym podejściu do zagadnienia rozwiązanie optymalne uzyskano poprzez zmniejszenie zmiennej stanu zadania o wartość iloczynu przyjętych arbitralnie tolerancji i odpowiednich wrażliwości tej zmiennej stanu. Metodę rozwiązania zilustrowano przykładami ściskania jedno- i dwuosiowego płyty prostokątnej, swobodnie podpartej czterostronnie. Jako kryterium optymalizacji przyjęto minimum ciężaru (grubości) płyty. Zapisano rekurencyjne zależności na wrażliwość siły krytycznej względem grubości poszczególnych warstw laminatu. Następnie, na podstawie tych zależności, wyznaczono zmodyfikowaną wartość siły krytycznej obciążenia. Szczegółowo omówiono wpływ badanych tolerancji na układ i kolejność warstw w laminacie wielowarstwowym. Uzyskane wyniki w pełni potwierdzają zasadność stosowania optymalizacji odpornościowej jako metody projektowania gwarantującej lepsze (bezpieczniejsze) rozwiązania niż standardowe ujęcie nominalne zagadnienia.

Słowa kluczowe: **płyta kompozytowa, optymalizacja, tolerancje wykonania, stabilność konstrukcji, optymalizacja odpornościowa**

INTRODUCTION

Laminated composites are commonly used structural materials, primarily due to their excellent strength and stiffness to weight ratios. However, composite material properties essentially depend on manufacturing processes and technology regimes, including proper temperature, pressure, curing time, specific prepreg storage and handling conditions etc. All these factors might be a source of potential error, resulting in unexpected deterioration of the composite mechanical and physical properties. This is especially important in the case of optimized composite structures, since optimum systems exhibit high sensitivity to variations in design variables. Considering the above-mentioned circumstances, robust design techniques are introduced to avoid manufacturing error-prone designs and to ensure a high-quality final product. Generally speaking, these techniques can be categorized into one of the three following concepts: stochastic analysis, interval programming and deterministic analysis.

The idea of the stochastic based approach to laminate stability estimation has been presented in numerous papers, e.g. by Kogiso et al. [1], and Singh et al. [2]. Within this concept, reliability analysis is derived considering the dispersion of design variables treated as random variables. In the examples discussed in the above-mentioned papers, these discrepancies result from uncertain individual lamina properties, orientation angles and applied loads. To estimate the final design state variables (e.g. buckling load magnitude), appropriate statistical test functions, probability distributions and perturbation techniques are used.

As an alternative concept, interval programming takes into account only the boundaries of the dispersed parameters to model system uncertainty. With respect to composite materials, this method has been studied by e.g. Jiang et al. [3]. The authors formulated a problem to find laminate ply orientations to maximise the stiffness of a plate if the material properties are subject to variation. The problem was solved with an arbitrarily chosen accuracy of the dispersed parameters. The discussed approach makes uncertainty analysis computationally attractive, although the optimal solution strongly depends on the arbitrarily predetermined possibility level.

The third way to address robust design is the deterministic one. Research is aimed at more in-depth analysis of the considered system but also at developing approximate simplified models as well. For instance Fong et al. [4] suggested taking into account second order effects into the design code of composite materials. The presented examples prove a more accurate reflection of specimen behaviour and considerable simplification in the final calculations for robust design. Alternatively, Lee and Park [5] proposed a new objective function defined as a linear combination of mean and standard deviation of the original cost function. This enables a design to be insensitive with respect to design parameter discrepancies. The deterministic approach to the discussed robust design problem was also studied by Gutkowski and Latalski [6] and next by Latalski [7]. The authors suggest diminishing the nominal limit value of the state variable in design constraints by appropriate positive terms. They are the products of assumed arbitrary design tolerances and a vector of appropriate state variable sensitivities.

In the current paper, the author's robust design idea is applied to the minimum weight design of a multi-layered laminate plate subject to uni- and bi-axial compression. Individual lamina thickness tolerances are taken into account. Updated formulas for the buckling load factor in the composite optimization problem are given. Based on these, the impact of thickness variations on the optimal laminate stacking sequence solution and buckling mode shape in the minimum weight design is discussed in detail.

PROBLEM STATEMENT

Let us consider a rectangular $a \times b$ multilayered composite panel simply supported on all its edges - see Figure 1. It is supposed that the laminate consists of N plies in total, each of an equal nominal thickness t. In each ply k, fiber orientation is denoted by θ_k and it is limited to four possible angles 0°, 90°, and ±45° only. Additionally, one assumes that the sandwich plate is symmetric and balanced - i.e. the number of plies having +45° fibers is equal to the number of plies with a –45° angle. Compressive longitudinal stress resultants λN_x and λN_y are applied at the edges of the panel along normal directions, so there is no shear load component.

To consider a non-ideal composite material, it is assumed that the actual value of an individual lamina thickness of the laminate may be varied from its nominal dimension t_k - see Figure 1b. This variation corresponds to the manufacture accuracy and is represented by a supposed *a'priori* maximum allowable deviation Δt_k (lower or upper). In further numerical analysis, this tolerance is set up arbitrarily as a ratio of nominal lamina thickness t_k. Therefore, if the upper and lower deviations stay the same, it is expected that the actual thickness of every k-th ply is within the range from $(t_k - \Delta t_k)$ to $(t_k + \Delta t_k)$.

In the present research, the stacking sequence of the minimum number of laminas assuring panel stability against buckling is sought. This is guaranteed by the constraint imposed on the load buckling factor λ (Fig. 1 and eq. (2)) to be greater or equal to one.

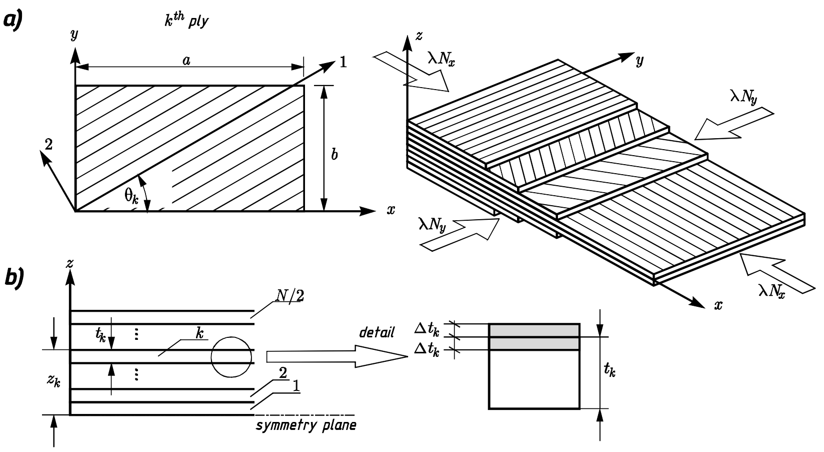

Fig. 1. Multilayered composite plate

Rys. 1. Wielowarstwowa płyta kompozytowa

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Allowing for manufacturing tolerances, where only the maximal acceptable deviations of design variables are given, indicates that their exact values are unknown. To deal with that problem, the following approach to optimum design is proposed. The optimization equality constraints are solved for nominal (average) values of design variables and imperfections (manufacturing tolerances) are introduced into the inequality constraints imposed on the state variables. This is done by diminishing their limit values by the product of design admissible imperfection and appropriate sensitivity variables - see more detailed discussion on this approach in [6].

Thickness tolerances impact and sensitivity analysis

According to the proposed solution of the robust design problem and discussion given in the above paragraph, the exact magnitude of the system performance criteria λ is unknown. Therefore it needs to be estimated by calculating the state variable λ_{cr} for nominal values of the design parameters and next it has to be diminished by penalty term $\Delta\lambda$, which is a product of thickness admissible imperfections Δt and appropriate sensitivities $d\lambda/dt$. Since the actual deviations of laminae thickness might be of any sign (upper or lower ones) and also the sensitivities might be of a positive or negative sign, the moduli of successive $\Delta t_k \cdot d\lambda/dt_k$ products are introduced into the resultant summation over all the laminate layers

$$\Delta\lambda = \sum_{k=1}^{N} \left| \frac{d\lambda}{dt_k} \Delta t_k \right| \tag{1}$$

This treatment ensures that the design stays on the safe side.

Finally, the obtained difference of the nominal system performance and the mentioned penalty term

$$\lambda_{cr}(m,n) - \sum_{k=1}^{N/2} \left| \Delta t_k \frac{d\lambda_{cr}}{dt_k} \right| \tag{2}$$

is considered to be a sufficiently accurate approximation of an imperfect system performance. As shown in previous research papers by Gutkowski and Bauer [8] and Gutkowski and Latalski [9], the use of the penalty term as proposed above results in more effective solutions while compared to simple worst case designs, where system safety is ensured by considering the structure with all nominal design variables t_k arbitrarily increased by the Δt_k summand.

The plate load factor λ_{cr} is evaluated according to the classical theory for a simply supported equivalent orthotropic plate subject to in-plane loading - see e.g. [10, 11]:

$$\lambda_{cr}(m,n) = \frac{D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(a/b)^2 + D_{22}n^4(a/b)^4}{m^2N_x + n^2(a/b)^2N_y} \tag{3}$$

where m and n are natural numbers corresponding to the number of buckling half-waves in the x and y directions respectively and variables D_{11}, D_{12}, D_{22} and D_{66} are the flexural stiffnesses. These four factors can be expressed in terms of material invariants U_i (i = 1, ..., 5) and three integrals V_0, V_1, V_3, comprising information about fiber orientations, about layer thicknesses and their stacking sequence - see e.g. [12]:

$$\begin{aligned} D_{11} &= U_1V_0 + U_2V_1 + U_3V_3, \; D_{12} = U_4V_0 - U_3V_3 \\ D_{22} &= U_1V_0 - U_2V_1 + U_3V_3, \; D_{66} = U_5V_0 - U_3V_3 \end{aligned} \tag{4}$$

Calculating the sensitivity factor necessary for the penalty term (1) one arrives at the vector

$$\left\{ \frac{d\lambda_{cr}}{dt_1}, \ldots, \frac{d\lambda_{cr}}{dt_N} \right\}^{\mathrm{T}}$$

were

$$\frac{d\lambda_{cr}}{dt_k} = \pi^2 \frac{\frac{dD_{11}}{dt_k} m^4 + 2\left(\frac{dD_{12}}{dt_k} + 2\frac{dD_{66}}{dt_k} \right) m^2n^2(a/b)^2 + \frac{dD_{22}}{dt_k} n^4(a/b)^4}{m^2N_x + n^2(a/b)^2N_y} \tag{5}$$

According to (4) the appropriate derivatives of flexural stiffnesses are as follows:

$$\begin{aligned} \frac{dD_{11}}{dt_k} &= U_1\frac{dV_0}{dt_k} + U_2\frac{dV_1}{dt_k} + U_3\frac{dV_3}{dt_k}, \\ \frac{dD_{12}}{dt_k} &= U_4\frac{dV_0}{dt_k} - U_3\frac{dV_3}{dt_k}, \\ \frac{dD_{22}}{dt_k} &= U_1\frac{dV_0}{dt_k} - U_2\frac{dV_1}{dt_k} + U_3\frac{dV_3}{dt_k}, \\ \frac{dD_{66}}{dt_k} &= U_5\frac{dV_0}{dt_k} - U_3\frac{dV_3}{dt_k} \end{aligned} \tag{6}$$

where individual terms dV_0/dt_k are constant

$$\frac{dV_0}{dt_k} = \frac{1}{2}h^2 \text{ for } k = 1,2,\ldots N \tag{7}$$

and dV_1/dt_k and dV_3/dt_k are given by the following recurrent formulas

$$\begin{aligned} &\frac{dV_1}{dt_1} = 2t^2\sum_{k=1}^{\frac{N}{2}}\left[k^2 - (k-1)^2\right]\cos 2\theta_k \; \frac{dV_3}{dt_1} \\ &= 2t^2\sum_{k=1}^{\frac{N}{2}}\left[k^2 - (k-1)^2\right]\cos 4\theta_k, \\ &\frac{dV_1}{dt_k} = \frac{dV_1}{dt_{k-1}} - 2(k-1)^2t^2(\cos 2\theta_{k-1} - \cos 2\theta_k) \\ &\text{for } k = 2,\ldots,N/2, \\ &\frac{dV_3}{dt_k} = \frac{dV_3}{dt_{k-1}} - 2(k-1)^2t^2(\cos 4\theta_{k-1} - \cos 4\theta_k) \\ &\text{for } k = 2,\ldots,N/2 \end{aligned} \tag{8}$$

Details concerning derivations of dV_0/dt_k, dV_1/dt_k and dV_3/dt_k terms are presented in paper [7].

NUMERICAL RESULTS AND DISCUSSION

Computations are performed for a plate having dimensions a = 50.8 cm and b = 25.4 cm (b/a = 0.5). Graphite-epoxy laminate material properties are assumed as follows: E_1 = 128.0 GPa, E_2 = 13.0 GPa, G_{12} = 6.4 GPa, nu_{12} = 0.3 and nominal ply thickness t = 0.127 mm. Axial load N_x is fixed at 5250 N/m and transverse load N_y is varied stepwise from 0 up to 13140 N/m. Thickness manufacturing tolerances are assumed to be given as a ratio of the nominal design value t - successively 1, 2, 3, 5, and 10%. All the calculations were performed by self-developed software; the code was written in C++ programming language.

First, uniaxial loading case studies were performed. It is known from the literature (e.g. [12]) that for the nominal design of plates with an aspect ratio a/b of more than about 0.7, an optimum solution consists of (±45°) plies only. Other orientations are possible only if the total number of layers is not dividable by 4, since this would lead to the violation of structural balance and symmetry constraints. A check was performed to see whether this solution is repeated in the approach where thickness tolerances are taken into account. It was found that for the considered cases of tolerance magnitudes this remains true. For solutions with a minimal number of layers dividable by 4, solely ±45° plies are present (see Δt = 45°), while for the remaining N values the innermost layer is set exchangeably to a 0°/90° angle. This exact location results from the fact that at this position lower stiffness properties of a ply are less significant for overall plate stability when compared to distal positions.

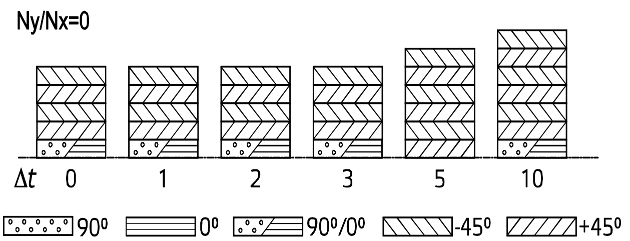

Fig. 2. Laminate stacking sequences in optimum designs for different thickness tolerances. Notation 90°/0° corresponds to 0° and 90° layers to be used interchangeably

Rys. 2. Ułożenie warstw laminatu w rozwiązaniach optymalnych dla różnych wartości tolerancji. Zapis 90°/0° oznacza warstwę albo 90°, albo zamiennie 0°

The outcomes of the analysis are given in Figure 2 and in Table 1. The number of observed equivalent solutions (N.s.) results directly from the simple ±45° stiffness symmetry. Moreover, it is worth mentioning that independent of the number of layers N and plies, the presence/absence of 0°/90° plies, the buckling mode is invariant.

TABLE 1. **Amplitude load factors and laminate stacking sequence for optimal solutions of uniaxial loading problem vs different manufacturing tolerance Δt magnitudes. N.s. - number of equivalent solutions**

TABELA 1. **Wartość współczynnika amplitudy obciążeń i układy warstw kompozytu dla różnych wartości tolerancji Δt. N.s. oznacza liczbę alternatywnych rozwiązań równoważnych**

Δt	λ	Stacking sequence	(m,n)	N.s.
0	1.411	$(90°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
	1.411	$(0°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
1	1.327	$(90°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
	1.327	$(0°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
2	1.242	$(90°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
	1.242	$(0°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
3	1.157	$(90°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
	1.157	$(0°, +45°, +45°, -45°, -45°)_s$	(2,1)	6
5	1.712	$(+45°, +45°, +45°, -45°, -45°, -45°)_s$	(2,1)	20
10	1.552	$(90°, +45°, +45°, +45°, -45°, -45°, -45°)_s$	(2,1)	20
	1.552	$(0°, +45°, +45°, +45°, -45°, -45°, -45°)_s$	(2,1)	20

Next, the biaxial loading cases were solved; the results are presented in Figure 3 and Table 2. Examples of nominal design given in the literature report that additional transversal loading leads to solutions containing 90° layers; the higher the N_y/N_x ratio, the more 90° laminas present at the cost of ±45° ones. This observation remains in force also for robust designs while the total number of layers does not change. In cases where the considered tolerances force new layers to be introduced, the additional ones are either ±45° or 90° plies. The first possibility happens for low N_y/N_x load ratio and low tolerance limits; obviously an additional condition is the updated number of layers to be divisible by 4 due to the symmetry and balance constraints. The case of additional 90° layers is observed for high load ratios and/or higher tolerances - see examples of $N_y/N_x \geq$ 1.5 to 2.5. It is often related to the substitution of a 0° or 90° layer by a solely 90° one. Moreover, an increase in load ratio N_y/N_x for a fixed tolerance level causes the existing ±45° layers to be placed closer to the laminate symmetry plane. Extrapolating the given results, one can conclude that a further increase in N_y/N_x load ratio would lead to purely 90° solutions only.

Another interesting observation deals with the total number of optimum solutions. This number is related not only to the simple symmetry of stiffness properties for +45° and –45° fiber orientations as initially expected and due to the added layers as discussed in the paragraph above. For the tolerance level Δt = 10% and most of the examined loadings ratios, completely new solutions exhibiting different stacking sequence orders are found. Special attention must be paid to the N_y/N_x = 1 case, where even twenty different stacking sequences are equivalent - see Table 2. Any of them can be assigned to one of the four groups (a–d) as depicted in Figure 3.

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TABLE 2. **Amplitude load factors and mode shapes for optimal solutions of biaxial loading problem vs different manufacturing tolerance Δt magnitudes. If alternative solutions are present (see Figure 3), they are denoted as cases 10^{a-d} respectively**

TABELA 2. **Wartość współczynnika amplitudy obciążeń i układy warstw kompozytu dla różnych wartości tolerancji Δt w przypadku ściskania dwuosiowego. N.s. oznacza liczbę alternatywnych rozwiązań (równoważnych). Ewentualne rozwiązania alternatywne (patrz rys. 3) oznaczono 10^{a-d}**

Load: N_y/N_x = 0.25			
Δt	λ	(m,n)	N.s.
0	1.039	(2,1)	2
1	1.689	(1,1)	6
2	1.581	(1,1)	6
3	1.473	(1,1)	6
5	1.258	(1,1)	6
10	1.140	(1,1)	6

Load: N_y/N_x = 0.5			
Δt	λ	(m,n)	N.s.
0	1.325	(2,1)	4
1	1.245	(2,1)	4
2	1.166	(2,1)	4
3	1.086	(2,1)	4
5	1.479	(2,1)	6
10^a	1.262	(1,1)	6
10^b	1.262	(1,1)	6

Load: N_y/N_x = 1.0			
Δt	λ	(m,n)	N.s.
0	1.390	(2,1)	6
1	1.307	(2,1)	6
2	1.223	(2,1)	6
3	1.140	(2,1)	6
5	1.452	(1,1)	2
10^a	1.182	(1,1)	4
10^b	1.182	(1,1)	4
10^c	1.182	(1,1)	6
10^d	1.182	(1,1)	6

Load: N_y/N_x = 1.5			
Δt	λ	(m,n)	N.s.
0	1.036	(2,1)	2
1	1.452	(1,1)	2
2	1.360	(1,1)	2
3	1.267	(1,1)	2
5	1.082	(1,1)	2
10^a	1.208	(2,1)	4
10^b	1.208	(2,1)	2

Load: N_y/N_x = 2.0			
Δt	λ	(m,n)	N.s.
0	1.231	(2,1)	4
1	1.157	(2,1)	4
2	1.083	(2,1)	4
3	1.439	(2,1)	2
5	1.228	(2,1)	2
10	1.281	(2,1)	4

Load: N_y/N_x = 2.5			
Δt	λ	(m,n)	N.s.
0	1.023	(2,1)	2
1	1.369	(2,1)	2
2	1.282	(2,1)	2
3	1.195	(2,1)	2
5	1.020	(2,1)	2
10^a	1.064	(2,1)	2
10^b	1.064	(2,1)	2

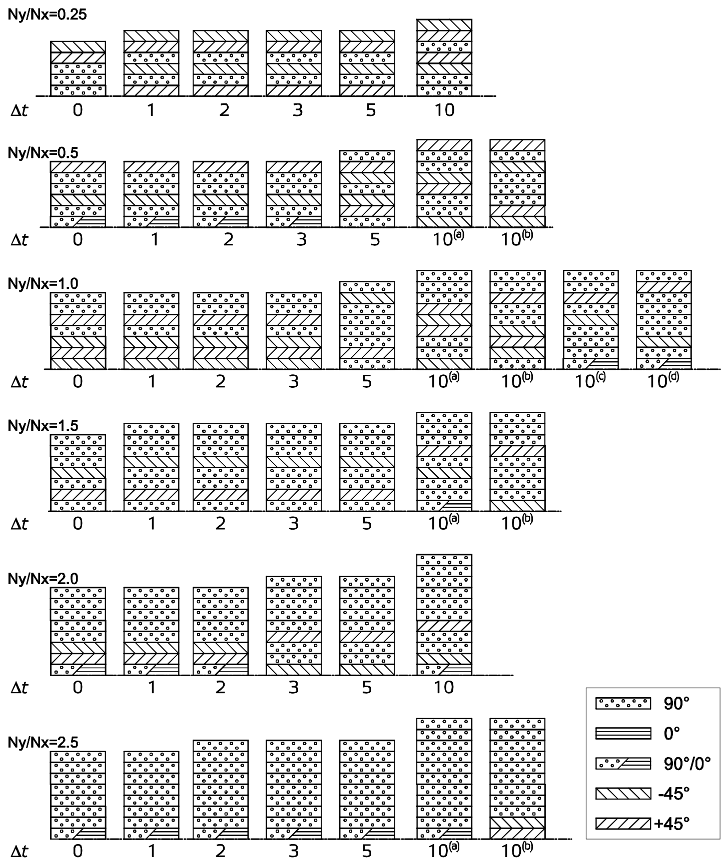

Fig. 3. Laminae stacking sequences in optimum designs for different thickness tolerances-biaxial plate compression cases. Notation 90°/0°corresponds to 0° and 90° layers to be used interchangeably. Alternative stacking sequence solutions for 10% tolerance are present and denoted by appropriate subscripts 'a' up to 'd'

Rys. 3. Ułożenie warstw kompozytu w rozwiązaniach optymalnych przy różnych wartościach tolerancji - ściskanie dwukierunkowe. Zapis 90°/0°oznacza warstwę albo 90°, albo zamiennie 0°. Alternatywne rozwiązania dla przypadku Δt = 10% oznaczone indeksami górnymi 'a' do 'd'

Further conclusions can be derived directly from Table 2. First, the observed variations in optimum stacking sequence solutions are also reflected by different buckling shapes exhibited by these results. While for nominal designs only (2,1) modes are expected, the robust solutions are both (2,1) as well as the (1,1) type. No specific regularity with respect to stacking sequence seems to exist. Moreover, in all the studied load cases, a high sensitivity of buckling load factor λ to tolerance magnitude is observed. In consequence of this feature, additional layers are necessary to satisfy the buckling constraint. This confirms the more general observation that any near-optimal solution is always very sensitive to design variables changes - see e.g. [13].

CONCLUDING REMARKS

The problem of the optimum design of composite laminates considering ply thickness tolerances has been addressed by the deterministic approach. Updated formulas for system performance and problem cost function have been derived based on classical laminated plate theory assumptions. Moreover, detailed sensitivity analysis has been performed. The cases of uniaxial and biaxial loading and different tolerance magnitudes have been discussed. It has been shown that for the uniaxial loading case, optimum stacking sequences mostly correspond to the solutions of the nominal design problem. The only observed changes are related to the additional outermost layers, which are required by the stability constraint. For biaxial loading, this observation has not been fully confirmed. It has been shown that for high tolerance levels and most of the examined loading ratios, entirely new solutions exhibiting a different stacking sequence order comparing to nominal designs are found. Besides that, it has been shown that for biaxial loading cases, changes in the buckling mode shape are expected with respect to the tolerance level.

The obtained results confirm the high sensitivity of the buckling load factor to tolerance magnitudes, for both uniaxial and biaxial cases. In consequence of this feature, additional layers are necessary to satisfy the buckling constraint even for relatively low tolerance levels. This observation in turn confirms the advantage of the derived approach to robust optimum design over deterministic, nominal methods reported in the literature.

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