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COMPOSITES THEORY AND PRACTICE

Daniel Nycz, Agnieszka Bondyra, Marian Klasztorny*, Paweł Gotowicki

[1] *Military University of Technology, Faculty of Mechanical Engineering, Department of Mechanics & Applied Computer Science*
ul. S. Kaliskiego 2, 00-908 Warsaw, Poland
**Corresponding author. E-mail: m.klasztorny@gmail.com*

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NUMERICAL MODELLING AND SIMULATION OF COMPOSITE SEGMENT BENDING TEST AND EXPERIMENTAL VALIDATION

The three-point bending test has been performed for a single-wave glass-polyester laminate segment. The geometry and stacking sequence of the segment is modelled on a selected composite tank cover. The main purpose of the study is to develop numerical modelling and simulation methodology for such a test using FE code MSC.Marc/Mentat as well as to perform experimental validation. The segment was manufactured in ROMA Ltd., Grabowic, Poland and made of a glass-polyester mixed laminate with a (CSM450/STR600)$_4$/CSM450 stacking sequence, using contact technology. The elastic and strength constants of the laminas have been derived experimentally on standard specimens cut from homogeneous laminates M (5xCSM450) and F (4xSTR600). The laminate components constitute: Polimal 104 polyester resin (matrix; produced by Organika-Sarzyna Co., Poland), E-glass mat and E-glass 1/1 plain weave fabric (reinforcement; produced by KROSGLASS Co., Poland). The numerical tests include the application of six selected shell finite elements, which accept layered composite material declaration, available in the MSC.Marc FE library. Simulated pressure force - punch displacement diagrams are presented against the respective experimental results. It has been pointed out that Element_75 (Bilinear Thick Shell) gives the results closest to reality, both qualitatively and quantitatively. A set of options/values of numerical modelling and simulation parameters elaborated by the authors' team in earlier papers has been applied. Index failures contours related to subsequent laminas are recommended for the design of shell segments of laminate covers of tanks and canals.

Keywords: **polymer-matrix composite cover, single-wave rectangular segment, glass-polyester laminate, bending test, numerical modelling, simulation, experimental validation, MSC.Marc system**

MODELOWANIE NUMERYCZNE I SYMULACJA PRÓBY ZGINANIA SEGMENTU KOMPOZYTOWEGO ORAZ WALIDACJA EKSPERYMENTALNA

Przeprowadzono próbę zginania „trójpunktowego" segmentu jednofalowego wykonanego z laminatu poliestrowo--szklanego, o geometrii i strukturze wzorowanej na wybranym przekryciu kompozytowym zbiornika. Celem pracy jest opracowanie metodyki modelowania numerycznego i symulacji takiej próby w środowisku obliczeń inżynierskich MSC.Marc/Mentat oraz walidacja eksperymentalna. Segment wytworzono w przedsiębiorstwie ROMA Sp. z o.o. z laminatu mieszanego o sekwencji warstw nośnych (CSM450/STR600)$_4$/CSM450. Stałe sprężystości i wytrzymałości lamin wyznaczono na próbkach wytworzonych z laminatów jednorodnych M (5xCSM450), F (4xSTR600). Komponenty laminatów są następujące: żywica poliestrowa Polimal 104 (osnowa; producent Organika-Sarzyna), mata ze szkła E oraz tkanina ze szkła E (wzmocnienie; tkanina o splocie prostym 1/1; producent KROSGLASS Krosno). Przetestowano przydatność sześciu wybranych elementów skończonych z biblioteki elementów w systemie MSC.Marc, dla których możliwa jest deklaracja kompozytu warstwowego. Wyniki symulacji przedstawiono na tle wyników eksperymentalnych. Wykazano, że Element_75 (Bilinear Thick Shell) prowadzi do wyników zgodnych jakościowo i ilościowo z wynikami eksperymentalnymi. W modelowaniu i symulacji zastosowano układ opcji/wartości parametrów modelowania i symulacji wypracowany przez autorów we wcześniejszych pracach. Do projektowania segmentów powłokowych przekryć zarekomendowano mapy indeksów niszczenia poszczególnych warstw laminatu.

Słowa kluczowe: **przekrycie kompozytowe, segment prostokątny jednofalowy, laminat poliestrowo-szklany, test zginania, modelowanie numeryczne, symulacja, walidacja eksperymentalna, system MSC.Marc**

INTRODUCTION

Shell structures (with or without stiffening interlayer) made of polymer-matrix laminates are increasingly more frequently applied in different industrial branches such as motorization, ship building, civil engineering, aviation or space technology. Such structures should be optimized by protecting respective safety and durability levels. Thus, the correct numerical modelling and simulation of static or dynamic engineering calculations of those structures has become of fundamental significance.

The basic approach in modelling shell laminas is homogenization, i.e. replacing a matrix - fibre heterogeneous system in the micro-scale with an equivalent material in the macro-scale. The equivalent body described by elastic and strength constants and respective constitute equations may be an iso-, mono- or orthotropic medium dependent on the reinforcement type. Homogenization is carried out using approximate methods that belong to the strength of materials, e.g. the mixed rule [1], numerical methods, e.g. [2, 3] or quasi-exact methods, e.g. [4]. The simplest and very accurate method constitutes experimental identification of the elastic-strength properties of respective homogeneous laminates reflecting laminas, e.g. [5].

Macro-mechanical models of layered composite shells are formulated in accordance to respective lamination theory, in which the laminate is made up of perfectly bonded layers. Each lamina is modelled by equivalent homogeneous material. In the case of 2D numerical models of layered shells, two main lamination theories groups can be distinguished ([6-9]), i.e. equivalent single-layer (ESL) theories and discrete-layer (DL) theories. In ESL theory, a laminate is represented by a single layer having micro-mechanical properties estimated as the weighted average values of subsequent lamina properties. The ESL model used in conjunction with the classic Kirchhoff - Love thin shell theory is commonly known as the classical laminate theory (CLT), e.g. [1]. The von Kármán-type CLT model useful for the analysis of geometrically non-linear laminate plates was developed by Sun & Chin [10], while the model for thin shells was formulated by Saidal et al. [11].

In order to predict the elastic behaviour of laminates with good accuracy, taking into consideration transverse shear deformation is obligatory. Improved theories hold the names of refined CLT theories for plates [12], shells [13] and sandwich shells [14]. These models require an appropriate transverse shear deformation factor due to constant shear distribution across the shell thickness, e.g. [15].

In discrete-layer (DL) theories, also termed as layer-wise formulations, each lamina is taken into account separately. The laminate is treated as a stack of laminas with separate DOFs, bounded together by appropriate conditions at the ply interfaces [9, 16-18].

The mathematical modelling of the statics of laminates and laminate sandwiches were collected and generalized by Kreja [15]. The writer considered geometrically non-linear elastic shells at small, medium and large rotations, under the following assumptions: large deformations with finite rotations and small strains, a smooth central surface of the shell, constant thickness of the shell during deformation and the 1st order shear deformation theory. For large deformation analyses, the writer recommends a Serendipita 8-node finite element with uniformly reduced integration.

Summing up, the literature review shows that the numerical modelling and simulation methodology of the response of a polymer-matrix laminate shell structure should be matched to the problem undertaken (statics, stability, dynamics) and to the assumptions (type and thickness of the shell, laminate type, non-linearities type, joints types, loadings etc.). Experimental validation and verification of numerical modelling and simulation is obligatory, performed on respective structural segments/systems.

DESCRIPTION OF COMPOSITE SEGMENT AND VALIDATION TEST

A single-wave polymer-matrix laminate segment of span length l = 4,00 m is examined experimentally. The segment is made of a mixed laminate with a stacking sequence modelled on a sewage-treatment plant cover ZABA Boehringer Ingelheim, Germany: gelcoat/ $(CSM450/STR600)_4$/CSM450/topcoat. The external protective layers have a 0.3 mm thickness, the mat-reinforced layers (CSM450) are of 1.06 thickness, and fabric-reinforced layers (STR600) have a 0.75 mm thickness. The laminate components constitute: Polimal 104 polyester resin (matrix; produced by Organika-Sarzyna Co., Poland), E-glass mat and E-glass 1/1 plain weave fabric (reinforcement; produced by KROS-GLASS Co., Poland). The elastic and strength constants describing the laminas has been derived experimentally on standard specimens cut from homogeneous laminates M (5xCSM450) and F (4xSTR600).

The three-point bending test for the laminate segment has been performed on a SATEC 1200 testing machine. The shape and overall dimensions of the segment are shown in Figure 1. The supports are made of IPE300 iron beams stiffened with 10 mm thick steel cross-plates. The top beam flanges are enforced with 400x170x20 mm plates fixed with M8 bolts. The loaded edge (during the test) of this plate was rounded off to a radius of 5 mm.

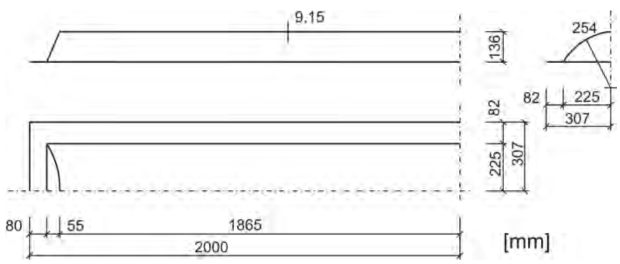

Fig. 1. Overall dimensions of composite segment quarter

Rys. 1. Wymiary gabarytowe 1/4 segmentu kompozytowego

The load was realized kinematically via the vertical motion of the machine punch at a constant velocity of 1 mm/s over a 0÷280 mm distance. The cubicoid iron punch had horizontal dimensions 470x190 mm. The static bending process was registered with a Phantom v12 video camera. The test stand is presented in Figure 2, whereas selected photos from the bending tests are shown in Fig. 3. The composite girder loses load capacity through gradual flattening of the composite shell central part up to local buckling and failure (Fig. 3).

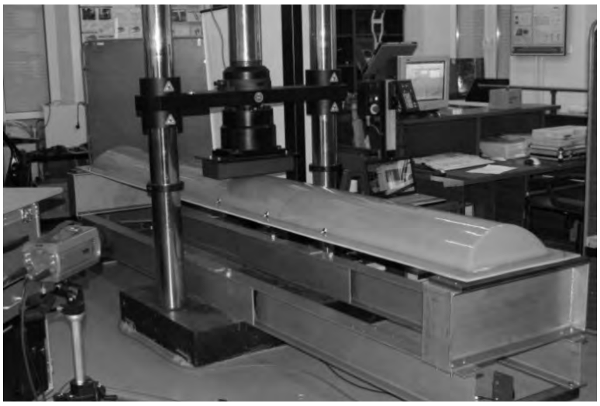

Fig. 2. Test stand for composite segment bending test

Rys. 2. Stanowisko do próby zginania segmentu kompozytowego

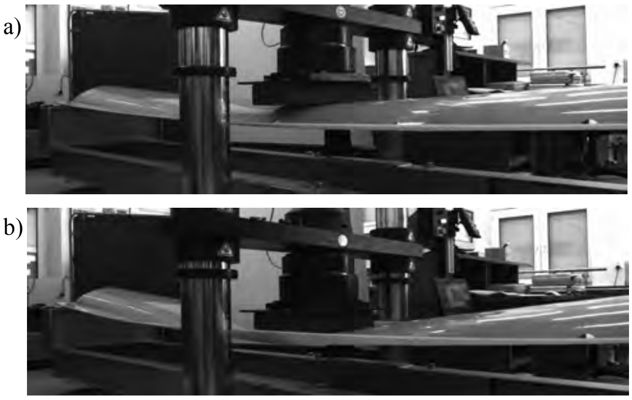

Fig. 3. Segment bending test at selected punch positions: a) s = 160 mm; b) s = 280 mm

Rys. 3. Próba zginania segmentu w pozycji: a) s = 160 mm; b) s = 280 mm

NUMERICAL MODELLING AND SIMULATION OF SHELL SEGMENT BENDING TEST

Owing to bisymmetry, the modelling can be limited to a quarter of the segment. The numerical modelling and simulation has been performed using MSC.Marc/ Mentat software. The numerical model of the segment has been constructed using the HyperMesh programme (Fig. 4).

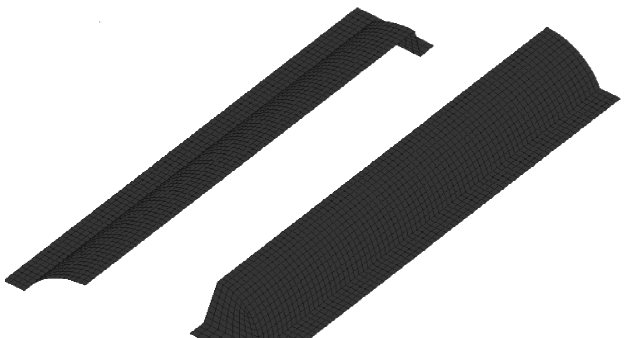

Fig. 4. FEM numerical model of composite segment quarter

Rys. 4. Model MES 1/4 segmentu powłokowego

Six shell elements, i.e. No. 22, 72, 75, 139, 149, 185, available in the MSC.Marc FE library [19] have been tested. For these elements, layered composite declaration is acceptable.

Subsequent laminas reinforced alternately with E-glass mat and fabric are modelled as linear elastic-short orthotropic materials, using material constants determined experimentally according to respective standard codes, collected in Table 1. The external protective layers (gelcoat, topcoat) are modelled as isotropic material close to Polimal 104 resin of parameters E = 3600 MPa; ν = 0.4; ε_f = 0,036; ρ = = $1.17 \cdot 10^{-9}$ t/mm^3 (Young's modulus, Poisson's ratio, ultimate tensile strain, density).

TABLE 1. **Elastic and strength constants of laminas reinforced with mat (M) and fabric (F)**

TABELA 1. **Stałe sprężystości i wytrzymałości lamin wzmocnionych matą (M) i tkaniną (F)**

Material constant	M	F	unit
Young's modulus E_1	8250	16550	[MPa]
Young's modulus E_2	8250	16550	[MPa]
Young's modulus E_3	4150	5000	[MPa]
Poisson's ratio ν_{12}	0.390	0.155	-
Poisson's ratio ν_{23}	0.235	0.234	-
Poisson's ratio ν_{31}	0.118	0.0707	-
shear modulus G_{12}	3200	2300	[MPa]
shear modulus G_{23}	3100	2400	[MPa]
shear modulus G_{31}	3100	2400	[MPa]
tensile strength X_t	95.7	269	[MPa]
compressive strength X_c	216	202	[MPa]
tensile strength Y_t	95.7	269	[MPa]
compressive strength Y_c	216	202	[MPa]
tensile strength Z_t	70	70	[MPa]
compressive strength Z_c	231	344	[MPa]
shear strength S_{xy}	91	32.6	[MPa]
shear strength S_{yz}	35.9	22.5	[MPa]
shear strength S_{zx}	35.9	22.5	[MPa]
ultimate normal strains at tension e_{1t}	0.021	0.021	-
ultimate normal strains at compression e_{1c}	0.031	0.011	-
ultimate normal strains at tension e_{2t}	0.021	0.021	-
ultimate normal strains at compression e_{2c}	0.031	0.011	-
ultimate normal strains at tension e_{3t}	0.017	0.020	-
ultimate normal strains at compression e_{3c}	0.061	0.100	-
ultimate shear strains g_{12}	0.043	0.050	-
ultimate shear strains g_{23}	0.040	0.045	-
ultimate shear strains g_{31}	0.040	0.045	-
friction ratio μ	0.29	0.29	-
density ρ	$1.42 \cdot 10^{-9}$	$1.68 \cdot 10^{-9}$	[t/mm^3]

For the E-glass mat reinforced homogeneous laminate, the Max Strain failure criterion has been chosen, for which the MSC.Marc system calculates 6 failure indices (IF1-IF6) at each integration point. The FI values belong to the 0-1 interval and are calculated from the following formulas [20]:

$$\begin{cases} \left(\frac{\varepsilon_1}{e_{1t}}\right) \text{ for } \varepsilon_1 > 0 \\ \left(-\frac{\varepsilon_1}{e_{1c}}\right) \text{ for } \varepsilon_1 < 0 \end{cases} \tag{1}$$

$$\begin{cases} \left(\frac{\varepsilon_2}{e_{2t}}\right) \text{ for } \varepsilon_2 > 0 \\ \left(-\frac{\varepsilon_2}{e_{2c}}\right) \text{ for } \varepsilon_2 < 0 \end{cases} \tag{2}$$

$$\begin{cases} \left(\frac{\varepsilon_3}{e_{3t}}\right) dla\ \varepsilon_3 > 0 \\ \left(-\frac{\varepsilon_3}{e_{3c}}\right) dla\ \varepsilon_3 < 0 \end{cases} \tag{3}$$

$$\left(\frac{\gamma_{12}}{g_{12}}\right) \tag{4}$$

$$\left(\frac{\gamma_{23}}{g_{23}}\right) \tag{5}$$

$$\left(\frac{\gamma_{31}}{g_{31}}\right) \tag{6}$$

where:
e_{1t}, e_{1c} - ultimate normal strain in direction 1, at tension and compression, respectively,
e_{2t}, e_{2c} - ultimate normal strain in direction 2, at tension and compression, respectively,
e_{3t}, e_{3c} - ultimate normal strain in direction 3, at tension and compression, respectively,
g_{12} - ultimate shear strain in plane 12,
g_{23} - ultimate shear strain in plane 23,
g_{31} - ultimate shear strain in plane 31,
with 1,2,3 - local coordinate system (1 - lamination axis, 3 - through-the-thickness axis).

For the E-glass fabric reinforced homogeneous laminate, the Hashin Fabric failure criterion has been selected, for which the MSC.Marc system also calculates 6 failure indices (IF1-IF6) at each integration point. The lamination angle equals 0. The FI values belong to the 0-1 interval and are calculated from the following formulas [20]:

$$\left[\left(\frac{\sigma_1}{X_t}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2\right] \text{ for } \sigma_1 > 0 \tag{7}$$

$$\left[\left(\frac{\sigma_1}{X_c}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2\right] \text{ for } \sigma_1 < 0 \tag{8}$$

$$\left[\left(\frac{\sigma_2}{Y_t}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2\right] \text{ for } \sigma_2 > 0 \tag{9}$$

$$\left[\left(\frac{\sigma_2}{Y_c}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2\right] \text{ for } \sigma_2 < 0 \tag{10}$$

$$\left[\left(\frac{\sigma_3}{Z_t}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2\right] \text{ for } \sigma_3 > 0 \tag{11}$$

$$\left[\left(\frac{\sigma_3}{Z_c}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13}}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2\right] \text{ for } \sigma_3 < 0 \tag{12}$$

where:
X_t, X_c - tensile/compressive strengths in the 1. direction,
Y_t, Y_c - tensile/compressive strengths in the 2. direction,
Z_t, Z_c - tensile/compressive strengths in the 3. direction,
S_{12} - shear strength in plane 12,
S_{23} - shear strength in plane 23,
S_{31} - shear strength in plane 31.

The material properties orientation of the finite elements has been declared in the Material Properties/ Orientations lap. The supports and loading punch have been modelled as rigid surfaces. Two frictionless planes of symmetry have been declared. The selective Gradual Degradation failure model has been applied. This model decreases the stiffness modules gradually during failure initiation. The touching (distance tolerance = 0.25, bias factor = 0.95) contact option has been chosen in the simulations. The Coulomb friction bilinear model (friction ratio μ = 0.2; ultimate normal stress σ_{lim} = 91 MPa) has been used. Dead load has been taken into account with the Gravity Load option. The problem has been solved using the full Newton - Raphson method as well as the displacement and force convergence criteria with 0,05 tolerance. The geometrical nonlinearity has been defined as large displacements, large rotations, and small strains.

NUMERICAL INVESTIGATIONS AND EXPERIMENTAL VALIDATION

A concise description of the analysed finite elements has been elaborated based on [19, 20]. Element_75 is a 4-node 2D bilinear finite element of the Thick Shell type, having 3 translational and 3 rotational DOFs at each node. The transverse shear strains are calculated at the central points of the edges and interpolated to the Gauss points. Element_22 is an 8-node 2D finite element of the Thick Shell type, having 3 translational and 3 rotational DOFs at each node. Second-rank interpolation is applied to coordinates, translations and rotations. Transverse shear strains are calculated at 10 special points and interpolated to the integration points. Element_139 is a 4-node 2D bilinear finite element of the Thin Shell type, having 3 translational and 3 rotational DOFs at each node.

Element_72 is an 8-node 2D shell finite element. At each corner node, translational DOFs with respect to the global Cartesian coordinate system are defined. At each intermediate node rotational, a DOF with respect to the element edge is introduced. Element_185 is an 8-node 3D finite element of the Solid Shell type, having 3 translational DOFs at each node. This element uses the assumed strain enforced formula for transverse strain components and assumed strain standard formula for shear components of the stress and strain tensors. In the case of layered material, each layer has three integration points.

Finally, Element _149 is an 8-th node 3D isoparametric finite element of a Composite Brick type. The number of layers must belong to the 2-510 interval. This element does not have a local coordinate system. The mechanical properties directions are specified in the material options: the first two axes of the local co-ordinate system are in-plane axes, while the last one is perpendicular to the lamina plane.

Figure 5 presents *F-s* curves (pressure force - punch position) corresponding to finite element types No. 22, 72, 75, 139, 149, 185, against the experimental results.

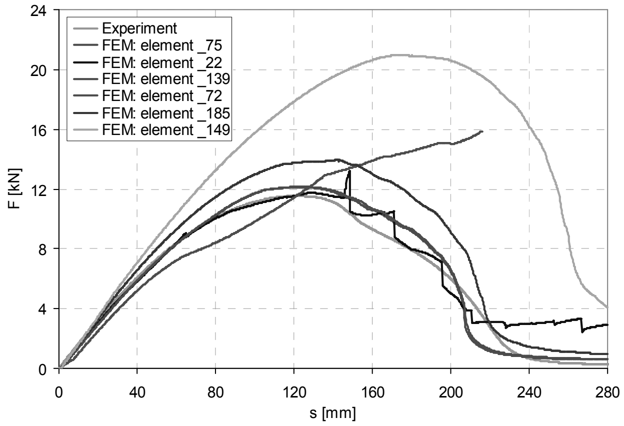

Fig. 5. *F-s* simulated curves against background of experimental result

Rys. 5. Porównanie przebiegów *F-s* symulacyjnych z przebiegiem eksperymentalnym

The simulated curves obtained for Element_75 (Bilinear Thick-shell Element) and Element_139 (Bilinear Thin-shell Element) are closest to reality. Owing to shear deformations taken into account, Element_75 is recommended for simulating static processes in laminate shells; the best result is presented separately in Figure 6. Figures 7, 8 present representative contours for vertical displacements and failure indices. The results in such a form can be applied to design composite covers or other laminate shell structures.

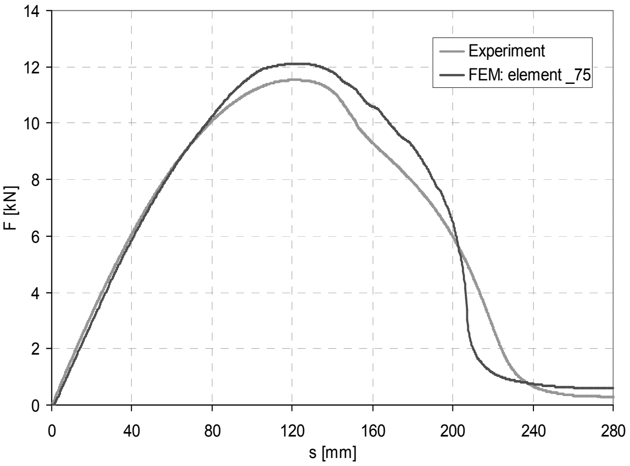

Fig. 6. Best simulated *F-s* result against background of experimental result

Rys. 6. Najlepszy wynik symulacyjny *F-s* na tle wyniku eksperymentalnego

Fig. 7. Vertical displacements contour *z*[mm] for s = 280 mm

Rys. 7. Mapa przemieszczeń pionowych *z*[mm] dla s = 280 mm

a)

b)

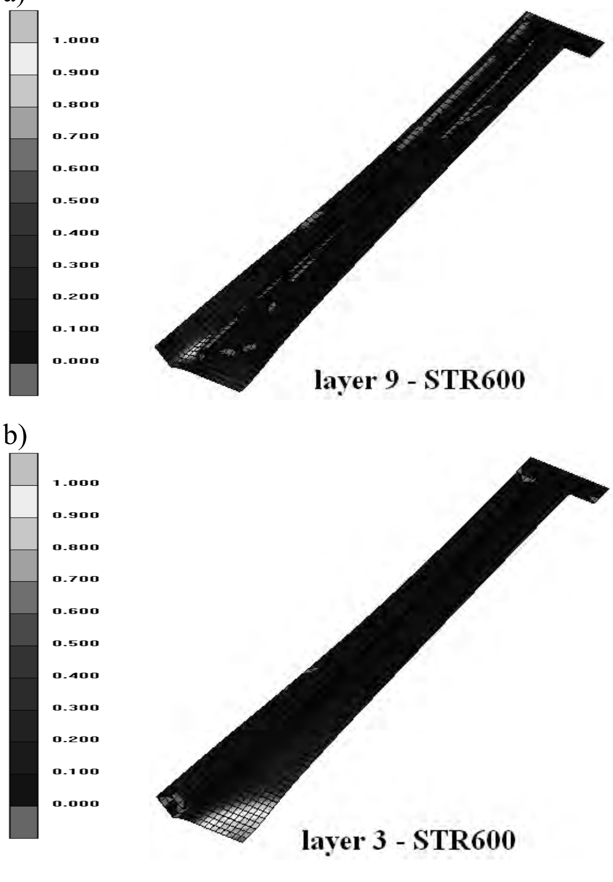

Fig. 8. Contours for selected failure indices for s = 280 mm: a) FI1 in layer 9; b) FI4 in layer 3

Rys. 8. Mapy wybranych indeksów niszczenia dla s = 280 mm: a) FI1 w warstwie 9; b) FI4 w warstwie 3

CONCLUSIONS

The study investigates the usability of six types of shell finite elements available in a MSC.Marc CAE system, for the numerical modelling and simulation of statics of shell girders made of polymer-matrix mixed laminates. Numerical investigations were conducted for a single-wave girder of a 4,00 m span length, under three-point bending up to local buckling and failure in the testing machine punch zone. A set of options/values of numerical modelling and simulation parameters established in previous papers of the authors' team was applied. It has been pointed out that Element_75 (Bilinear Thick-shell Element) leads to the results closest to the experimental results. This element corresponds to refined CLT theory. Using Element_75, the computations consume the shortest CPU time (1,3 hour). Because of the layered structure of the shell and orthotropy of the layers, the failure indices for respective failure criteria should be used for load capacity/safety coefficients assessment. Delamination phenomenon using Element_75 is approximately taken into consideration via the strength parameters taken from standard interlayer shear tests.

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